TECHNICAL REPORT NO. 11808

EVALUATION OF ACCURACY OF MEDIAN RANKS AND MEAN RANKS PLOTTING FOR RELIABILITY ESTIMATION USING THE WEIBULL DISTRIBUTION

June 1973

by S. B. Catalano
Materials Branch

TACOM

VEHICULAR COMPONENTS & MATERIALS LABORATORY

U.S. ARMY TANK AUTOMOTIVE COMMAND Warren, Michigan

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MATERIALS BRANCH

ABSTRACT

Estimates of the Weibull distribution parameters were made employing the mean ranks estimator; the estimates were repeated using the median ranks estimator. These estimates were compared to known values of the Weibull distribution parameters. This made it possible to compare the results obtained using either estimator (mean ranks or median ranks) and to determine the relative merits of using either estimator. This study made use of a digital computer and employed Monte-Carlo techniques to simulate Weibull distributed failure times. These failure times may represent tank-automotive component failures.

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INTRODUCTION

One of the earliest applications of the Weibull distribution in this country was in 1951 in a paper presented by Professor Weibull (1). Its use since then has been predominantly in the analysis of life test data in which the variable of interest is lifetime, t. The Weibull distribution is of interest to TACOM from the standpoint of analysis of life test data on tank-automotive components. The Weibull density function for the random variable, t, is:

$$f(t) = \frac{\beta}{\theta}\left(\frac{t - \alpha}{\theta}\right)^{-1} \exp\left[-\left(\frac{t - \alpha}{\theta}\right)^{\beta}\right] \text{ for } t \geq \alpha$$

$$f(t) = 0 \quad \text{for } t < \alpha.$$

where β is the shape parameter, or slope parameter (usually the value of β is near 3.5 for tank-automotive components; this means that the failure distribution curve has the familiar bell shape).

θ is the scale parameter, or characteristic life parameter (the units of θ for tank-automotive components are usually measured in miles or cycles till failure).

α is the location parameter, or minimum life parameter.

In general usage, $\alpha = 0$, in which case:

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right] \text{ for } t \geq 0$$

$$f(t) = 0 \quad \text{for } t < 0.$$

In this study we will be concerned with the cumulative Weibull distribution which is expressed mathematically as:

$$F(t) = 1 - \exp\left[-\left(\frac{t - \alpha}{\theta - \alpha}\right)^{\beta}\right],$$

or, if $\alpha = 0$:

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right].$$

In the application of the Weibull distribution in the analysis of life test data, use is made of estimators for the value of F(t). Estimates are necessary since life testing data yields only values of failure time t; the values of β and θ in equation (1) remain unknown. Two different estimators used for this purpose are called "median ranks" and "mean ranks" estimators.

Background information is presented in Appendix I. The computer program flow chart is shown in Figure 1. Figures 2 through 11 in Appendix I present various aspects of the Weibull distribution.

OBJECT

The objective of this study is to compare the accuracy of the two commonly used estimators called "median ranks" and "mean ranks" when employed in Weibull distribution failure analysis for estimating the Weibull parameters β and θ.

SUMMARY

Weibull distributed failure times were simulated on a computer via Monte-Carlo techniques. Using these simulated values of failure times, estimates of the Weibull parameters β and θ were computer calculated using the median ranks estimator; the calculations were repeated using the mean ranks estimator. The resulting estimates of β and θ were labeled $\hat{\beta}$ and $\hat{\theta}$ and were compared to the known values of β and θ that were used to simulate the Weibull distributed failures. In this manner, it was possible to compare the accuracy of median ranks and mean ranks estimators in calculating estimated values of Weibull parameters β and θ. The study was carried out for various sample sizes and for various values of the parameters β and θ; various degrees of suspended data were employed.

CONCLUSIONS

It is concluded from this study that use of the median ranks approximator provides a better approximation for $\hat{\theta}$ than mean ranks over the range of parameters employed in this study. This also tends to be the case for estimating the slope, $\hat{\beta}$, for cases where the number of test samples is high; this tendency is enhanced as the degree of suspended data is decreased.

The mean ranks estimator provides a better estimate of $\hat{\beta}$ than the median ranks estimator only in the region of small sample size and high degree of suspended data. But this region is a region which is inherently plagued with a high degree of error in estimating $\hat{\beta}$ regardless of which estimator is employed. The additional error that is encountered by using median ranks rather than mean ranks in this region is small in comparison with the error inherently encountered in this region.

RECOMMENDATIONS

It is deemed advantageous to routinely employ the median approximator when estimating the Weibull parameters β and θ. The slight loss of advantage in the region of small sample size and large degree of suspended data in the case of estimating $\hat{\beta}$ can either be corrected for by reference to Figures 14 through 19 of Appendix IV or dismissed as being negligible.

TEST PROCEDURE

The intent of this study was to objectively compare the resulting values of $\hat{\beta}$ and $\hat{\theta}$ when using mean ranks with those values of $\hat{\beta}$ and $\hat{\theta}$ obtained when using median ranks, for both suspended data tests and non-suspended data tests. The comparison was made by measuring $\hat{\beta}$ and $\hat{\theta}$ in each of twenty separate simulated failure tests using first the mean ranks estimator, then using the median ranks estimator. The standard error statistic was employed as the measure of accuracy for comparison purposes. In this case, the mathematical expressions for standard error in the measurement of β and θ are respectively

$$\left[\sum(\hat{\beta}-\beta)^2/20\right]^{1/2} \text{ and } \left[\sum(\hat{\theta}-\theta)^2/20\right]^{1/2}.$$

This study was carried out for a wide range of values of the parameters β, θ and sample size, n, under suspended and non-suspended data testing. The values of β used were 0.5, 1, 2, 4 and 8. The values of θ used were 5,000, 10,000, 20,000, 40,000, 80,000 and 160,000 miles. Test sample sizes used were n = 5, 10, 20, 40, 80 and 160 samples. Tests were run for 60%, 40% and 0% (i.e. non-suspended) suspended data runs.

The entire study was a computer study. Failures were simulated on a digital computer using Monte-Carlo techniques. Rather than plotting the resulting points and drawing the best fit straight line through these data points by sight, a computer subroutine was used to compute the best

fit using least squares fitting. In this manner it is expected that the results obtained are objective and free of differences due to personal traits in line plotting by eye. The computer was then asked to compute $\hat{\beta}$ and $\hat{\theta}$ and then to compute and print the standard error $[\Sigma(\hat{\beta}-\beta)^2/20]^{1/2}$ and $[\Sigma(\hat{\theta}-\theta)^2/20]^{1/2}$ for each of the three degrees of suspended data tests employed and for each combination of parameters β, θ and n employed in this study.

The flow diagram of the computer program used in this study is shown in Figure 1. Each step in the flow diagram will be explained in detail. The computer program as listed just prior to computer execution is shown in Appendix II. The resulting printout is shown in Appendix III.

Weibull distributed failure times are simulated (via Monte-Carlo method) by evaluating equation (1) for t using random numbers from 0.0 to 1.0 for F(t). In real life, the selection of random numbers from 0.0 to 1.0 represents the random selection of test samples from the entire population of samples. The significance of the random numbers ranging from 0.0 to 1.0 is that the test samples selected may equally well be the first to fail, the last to fail or to fail at any time between the first and last failures. In other words, the values from 0.0 to 1.0 represent a ranking of the failures onto a percentage scale. In equation (1), F(t) represents this ranking. Evaluating equation (1) for t using the random numbers for values of F(t) has the effect of grouping or modulating the failure times such that they will be Weibull distributed random failure times. In other words, the failure times so generated simulate failure times that would occur for samples that fail according to the Weibull probability distribution.

The need for random numbers in this study is apparent. Random number generators are not available in all computers. The computer used in this study was a Control Data Corporation computer CDC Model 6600. It possesses a system function RANF(0) which provides a random number from 0.0 to 1.0 each time RANF(0) is requested by the program. The number of decimal places utilized in this study for RANF(0) was eight places.

In this study sets of random numbers are used. The CDC system function RANF(0) furnishes a new set of random numbers each time a new set is called for in a program. However, each time the program is resubmitted to the computer, it gives the exact same sets of numbers in exactly the same sequence from start to finish.

The first step in the program flow diagram indicates the calling for a set of n random numbers.

STEP NUMBER	OPERATION
1.	Call for set of n random numbers
2.	Rank order the set of n random numbers
3.	Generate n Monte-Carlo Simulated Failure Times
4.	Generate n Median ranks (rank ordered)
5.	Generate n Mean ranks (rank ordered)
6.	Perform transformation of failure times to Weibull Probability Axes
7.	Perform transformation of mean ranks to Weibull Probability Axes
8.	Perform transformation of median ranks to Weibull Probability Axes
9.	Pair the n transformed failure times with the n transformed mean ranks
10.	Pair the n transformed failure times with the n transformed median ranks
11.	Determine $\hat{\beta}$ and $\hat{\theta}$ for mean ranks using least squares fitting subroutine
12.	Determine $\hat{\beta}$ and $\hat{\theta}$ for median ranks using least squares fitting subroutine
13.	Repeat steps 1 thru 12 M times (where M = 20)
14.	Determine standard errors $\left[\sum(\hat{\beta}-\beta)^2/M\right]^{1/2}$ & $\left[\sum(\hat{\theta}-\theta)^2/M\right]^{1/2}$ for mean ranks
15.	Determine standard errors $\left[\sum(\hat{\beta}-\beta)^2/M\right]^{1/2}$ & $\left[\sum(\hat{\theta}-\theta)^2/M\right]^{1/2}$ for median ranks
16.	Change to next value of n and repeat steps 1-15
17.	Change to next value of β and repeat steps 1-16
18.	Change to next value of θ and repeat steps 1-17

FIGURE 1. Computer Program Flow Chart

The second step in the program flow diagram shows that the set of n random numbers generated in the previous step will be rank ordered. The ordering is from lowest number to highest number. The random numbers are then used in this order to generate the simulated failure times. The resulting failure times are thus generated in rank (numerical or chronological) order. Aside from it being psychologically satisfying to have the failure times occurring in chronological order, thereby giving the simulation a real-life flavor, it facilitates pairing of failure times with the proper median/mean ranks: the first failure time with the first value of median/mean ranks; the second failure time with the second value of median/mean ranks, etc., till the last failure time is paired with the last value of median/mean ranks. This pairing is for purposes of forming coordinate points to be plotted/fitted onto Weibull probability coordinate axes. Note that it is immaterial whether the rank ordering step is performed before or after the generation of failure times or for that matter whether it's done at all. What is important is that somehow the j^{th} failure time is paired with the j^{th} median/mean ranks. Rank ordering facilitates this pairing.

The third, fourth and fifth steps in the program flow diagram indicate the generation of a set of n rank ordered failure times, median ranks and mean ranks respectively.

Up to this point, the computer has generated a set of n rank ordered random numbers, a set of n rank ordered failure times, a set of n rank ordered median ranks and a set of n rank ordered mean ranks. The next step is to perform a transformation of axes on the above sets of numbers to Weibull probability axes, (X, Y). The failure times, t, are transformed according to equation (6) of Appendix I. Since the random numbers represent various values of F(t) and the median ranks and the mean ranks are estimates of F(t), these three sets are transformed according to equation (5) of Appendix I. Steps 6, 7 and 8 of the program flow diagram indicate these transformations of axes.

The next block of steps in the program flow diagram is for performing the least squares fitting of a straight line to the set of coordinate points on Weibull probability coordinate axes. The first step in this process is to pair the j^{th} transformed failure times with: 1) the j^{th} median ranks, and 2) with the j^{th} mean ranks; this is shown in steps 9 and 10. The next step is to perform the actual least squares fitting. This is done with the subroutine LSFIT. It makes use of the least squares fitting procedure as outlined by Cullity (2). Accordingly, subroutine LSFIT computes the first and second normal equations for the simulated points on the Weibull probability coordinate axes. These two equations are linear equations; simultaneous solution yields the slope of the straight line and the Y-intercept. In this study, the slope is $\hat{\beta}$ and the

Y-intercept is $-\hat{\beta}\ln\hat{\theta}$ from which the value of $\hat{\theta}$ is readily obtained. Solution of the simultaneous equations is done in the main program. Steps 11 and 12 of the program flow diagram indicate computation of $\hat{\beta}$ and $\hat{\theta}$ for the median and mean ranks.

The next step in the program flow diagram indicates that all of the previous steps are to be repeated 20 times. In real life, this would simulate repeating 20 times the entire process of randomly selecting n test samples to be life tested, plotting the failure times (on Weibull probability paper) vs.: 1) mean ranks, and 2) median ranks, and then drawing the best fit straight line for both cases and determining $\hat{\beta}$ and $\hat{\theta}$ from the slopes and intercepts of these lines. This results in 20 values of $\hat{\beta}$ and $\hat{\theta}$ for mean ranks case and 20 for the median ranks case.

In the next step shown in the flow diagram, the program computes the standard errors $[\Sigma(\hat{\beta}-\beta)^2/20]^{1/2}$ and $[\Sigma(\hat{\theta}-\theta)^2/20]^{1/2}$ from the 20 values of $\hat{\beta}$ and $\hat{\theta}$ arrived at in the previous step. This is done for both the median ranks and mean ranks values of $\hat{\beta}$ and $\hat{\theta}$, and for each of the three degrees of suspended data (60%, 40% and 0%). This calculation is performed by the standard error subroutine MDEV listed at the end of the program.

As a reminder, it should be noted here that the values of β and θ are known. They were used early in the program in equation (1) to generate the simulated failure times.

The next three steps in the program direct the computer to repeat the computation for new parameter values, changing the value of one parameter at a time until all possible combinations of parameter values of β, θ and n used in this study have been used.

RESULTS AND DISCUSSION

The numerical values of standard errors $[\Sigma(\hat{\beta}-\beta)^2/20]^{1/2}$ and $[\Sigma(\hat{\theta}-\theta)^2/20]^{1/2}$ as calculated and printed by the computer are shown in Appendix III. These values of standard error are tabulated according to degree of suspended data (i.e. 60%, 40% or 0%) and are grouped according to parameter values of β and θ, each group making use of each value of n (sample size) employed in this study. There are 2,160 entries of standard error charted, thus making it overwhelmingly difficult to interpret simply by comparing numerical values. For this reason, these values of standard error were plotted; they were plotted vs. sample size, n. These plots are shown in Appendix IV (Figures 14 thru 25). Each figure contains plots for 60%, 40% and 0% suspended data thus facilitating interpretation

of results as a function of degree of suspended data. These plots are drawn for each of the five values of β used in this study and for both estimators (median ranks and mean ranks). In this manner, the results can be more readily interpreted as a function of β as well as a function of estimator employed. Only one value of θ is used per figure. Interpretations as a function of θ must therefore be made by making comparisons between the various figures.

In the cases of 60% and 40% suspended data, the plots do not include the points for n = 5 since in these cases, it would mean that the tests were terminated after only two and three samples, respectively, had failed, which is much too small a number of failures from which to obtain meaningful results. Nevertheless, the computer was asked to perform these calculations. In most cases, the results so obtained were either off the scale of the graphs shown or larger than the largest number allowed to be printed by the computer program format (in this case, an asterisk is shown in the computer printout), thereby preventing plotting of such points.

Figures 14 through 19 of Appendix IV are plots of standard error encountered in calculating $\hat{\beta}$; figures 20 through 25 of Appendix IV are for standard error encountered in calculating $\hat{\theta}$.

In general, the two effects that these graphs make most apparent are effects that are intuitively expected; these are that:

1. Standard error decreases with increasing sample size.

2. Standard error increases with increasing degree (or percentage) of suspended data.

Another effect that is immediately apparent from these graphs in the case of standard errors encountered in calculating $\hat{\beta}$ (see Figures 14 thru 19) is that standard error increases as β increases. It is also seen in this case that θ has little or no effect on standard error. However, in the case of standard errors encountered in calculating $\hat{\theta}$ (see Figures 20 thru 25), standard error decreases as β increases, and is significantly increased as θ is increased.

The zig-zagged appearance of the plots is due to connecting the plotted points with straight lines. Generally, the plots exhibit decreasing standard error with increasing sample size n, however, occasionally a point will deviate from this trend and will be higher than the previous point, when it was expected to be lower than the previous point. This possibly is caused by random sampling; if so, it is a reflection of the fluctuations experienced in real life failure testing when test samples are randomly chosen from the entire population of samples. This real life "flavor" is reflected into the results of this simulation study since a random number generator was used in generating Weibull distributed failure times.

Solid lines in the plots are used to illustrate the results obtained when using the median ranks estimator; broken lines are used for the results obtained when using the mean ranks estimator.

The following observations are made (relative to the solid vs. broken lines) in the case of standard errors encountered in calculating $\hat{\beta}$ (refer to Figures 14 thru 19):

1. Sometimes the solid lines are above the broken lines; sometimes they are below. Sometimes the two are overlapping.

2. The broken lines tend to be lower than the solid lines for lower values of n, and tend to be higher for higher values of n.

3. The solid lines tend to be lower than the broken lines as β increases and as the degree (or percentage) of suspended data is decreased.

4. The solid lines cross or touch the broken lines at least once in the range of values of n used in this study.

The following observations are made (relative to the solid vs. broken lines) in the case of standard errors encountered in calculating $\hat{\theta}$ (refer to Figures 20 thru 25):

1. The solid lines are either below the broken lines or the two are overlapping. In most cases, even when the two lines are shown overlapping, reference to the numerical values of the plotted points reveals that the solid line has the lower value at that point. There were several exceptions to this. However, in these cases the two values were so close numerically that it was not possible to depict the difference when plotting these values.

2. The solid and broken lines tended to be closer together as 1) β increased, 2) the degree of suspended data decreased, and 3) n increased.

As pointed out in Appendix I of this report, L. G. Johnson prefers using median ranks rather than using mean ranks when drawing the best fit line through the data points on Weibull probability paper by eye. He points out that by using median ranks one avoids underestimating the slope, $\hat{\beta}$, of the line in the region of the lower extreme of the graph (i.e. in the region where only relatively few of the test samples have failed). In terms of degree of suspended data, this corresponds to the region of higher degree (or percentage) of suspended data.

The results of this study indicate that the tendency to err in the estimate of slope, $\hat{\beta}$, at the higher degrees of suspended data tends to vary with n, the test sample size. The tendency is that for larger sample sizes, median ranks are more accurate than mean ranks, whereas for smaller sample sizes, the reverse is true.

Although Mr. Johnson did not expressly discuss the relative merits of using median ranks vs. mean ranks when estimating $\hat{\theta}$, one must assume that he prefers median ranks in this case also, since an error in slope, $\hat{\beta}$, would mathematically reflect an error in the estimate of $\hat{\theta}$. The results in this respect in the present study indicate that use of median ranks is as accurate as, or more accurate, than use of mean ranks.

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APPENDIX I

BACKGROUND

A short discussion of the general properties of the Weibull distribution follows:

The expected value of the Weibull distribution is

$$E(t) = \theta \Gamma\left(1 + \frac{1}{\beta}\right). \tag{2}$$

The variance of the Weibull distribution is

$$V(t) = \theta^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2\right]. \tag{3}$$

The shape of the Weibull density function

> "changes from a highly positively skewed distribution when β = 0.5, to a simple exponential distribution when β = 1, to an essentially Gaussian normal distribution, when β = 3.5, to a negatively skewed distribution when β = 6 or more." (3)

These various shapes are shown in Figure 2.

When $\beta = 1$,

$$f(t) = \frac{1}{\theta} \exp\left[-\frac{t}{\theta}\right],$$

which verifies that when β = 1, the Weibull distribution is equivalent to the simple exponential distribution. Figure 3 illustrates that when β = 3.5, the Weibull distribution is essentially a Gaussian normal distribution. This illustration was made by comparing a plot of the standard normal curve with a plot of the Weibull distribution having β = 3.5 and variance equal unity. Note that α = 0 in this plot of the Weibull distribution, and that the plot would "slide" to a new position along the horizontal axis as various non-zero values of α are used. Figure 3 also illustrates a plot of the Weibull distribution for β = 3.5 with variance equal 4 (i.e. standard deviation = 2).

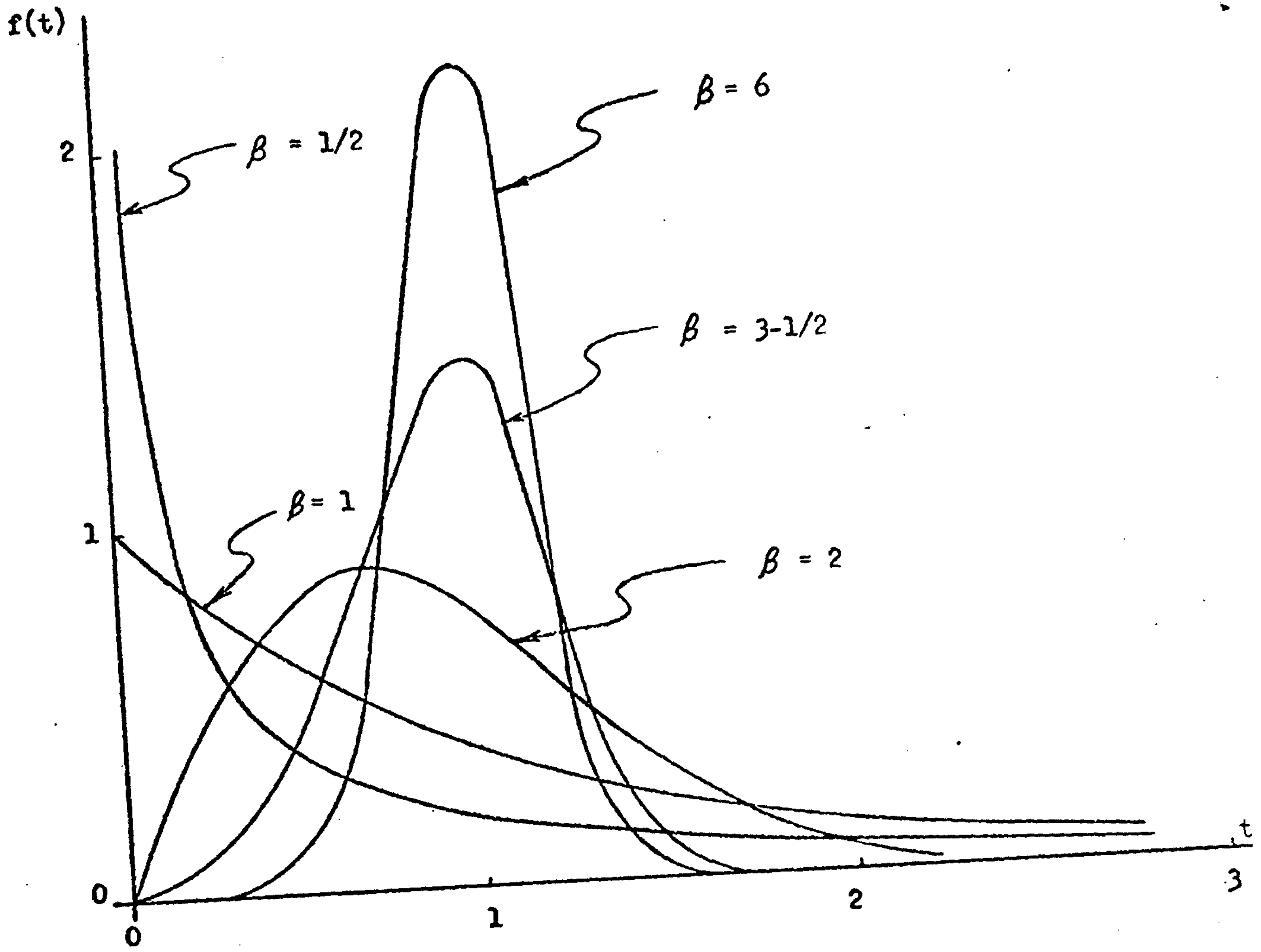


FIGURE 2. The various shapes of the Weibull Probability Distribution for various values of β.

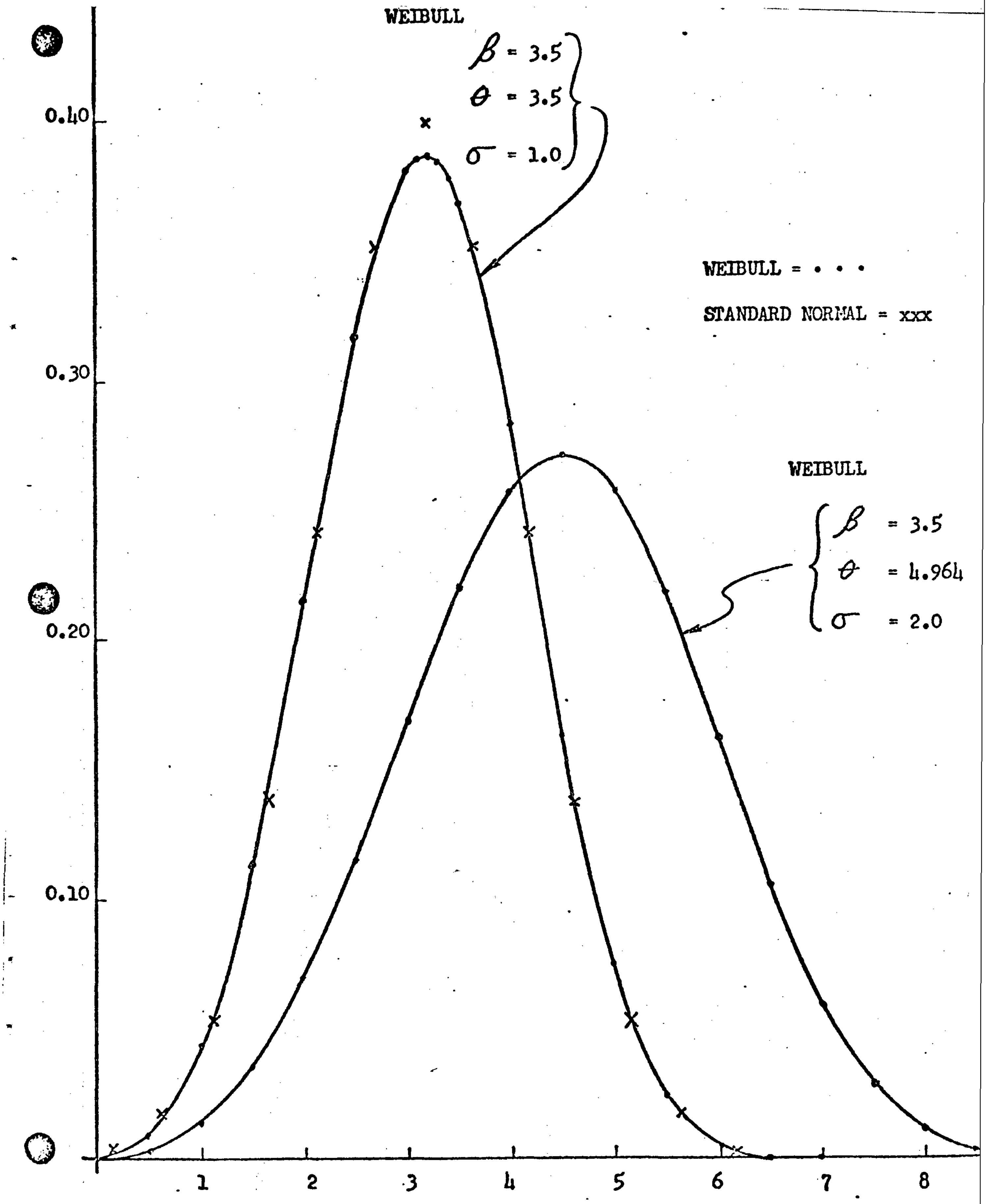

WEIBULL
β = 3.5
θ = 3.5
σ = 1.0
WEIBULL = • • •
STANDARD NORMAL = xxx
WEIBULL
β = 3.5
θ = 4.964
σ = 2.0
0.40
0.30
0.20
0.10
1
2
3
4
5
6
7
8

FIGURE 3

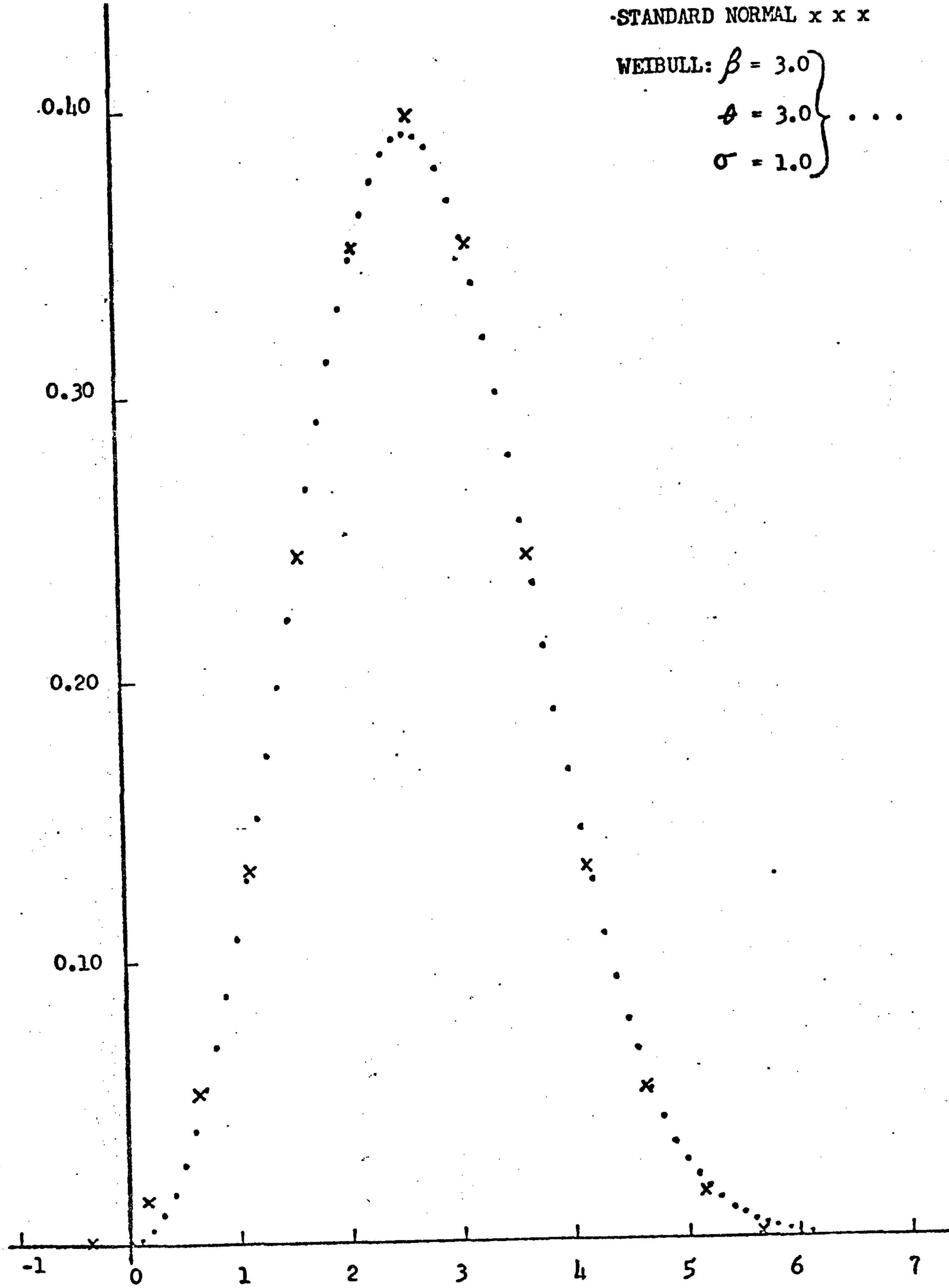
Figures 4 through 8 are plots of the Weibull distribution that were made using values of β equal to 3.0, 3.3, 3.5, 3.7 and 4.0. In each case the variance equals unity. For comparison purposes a standard normal curve is also plotted on each figure. This series of plots was made to verify that the Weibull distribution having $\beta = 3.5$ conforms closest to the standard normal curve. It is seen from these figures that of those values of β used, the Weibull distribution having $\beta = 3.5$ conforms closest to the standard normal curve. The greatest discrepancy here between the two distributions is observed to be at the peaks of the distributions. Nevertheless, it is apparent that the Weibull distribution having $\beta = 3.5$ and $\sigma = 1$ is a good approximation to the standard normal distribution.

Figures 9, 10 and 11 illustrate that the plot of the Weibull distribution becomes taller, slimmer and more negatively skewed as β gets larger. In each of these plots, the variance equals unity; the values of β in Figures 9, 10 and 11 are respectively 10, 40 and 100.

From equations (2) and (3), it is seen that as β approaches infinity, the expected value equals θ and that θ equals infinity. It appears that the Weibull distribution is of limited use at the larger values of β (such as $\beta = 100$). Its usefulness lies primarily in the region of the lower values of β, especially at $\beta = 1$, where the Weibull distribution is equivalent to the exponential distribution, and at $\beta = 3.5$ where it is essentially equivalent to the Gaussian normal distribution. Both the exponential distribution and the Gaussian distribution are useful in failure analysis. Consequently the Weibull distribution, due to its versatility, is very useful in failure analysis. In fact, its usefulness transcends that of either the exponential or Gaussian distribution since

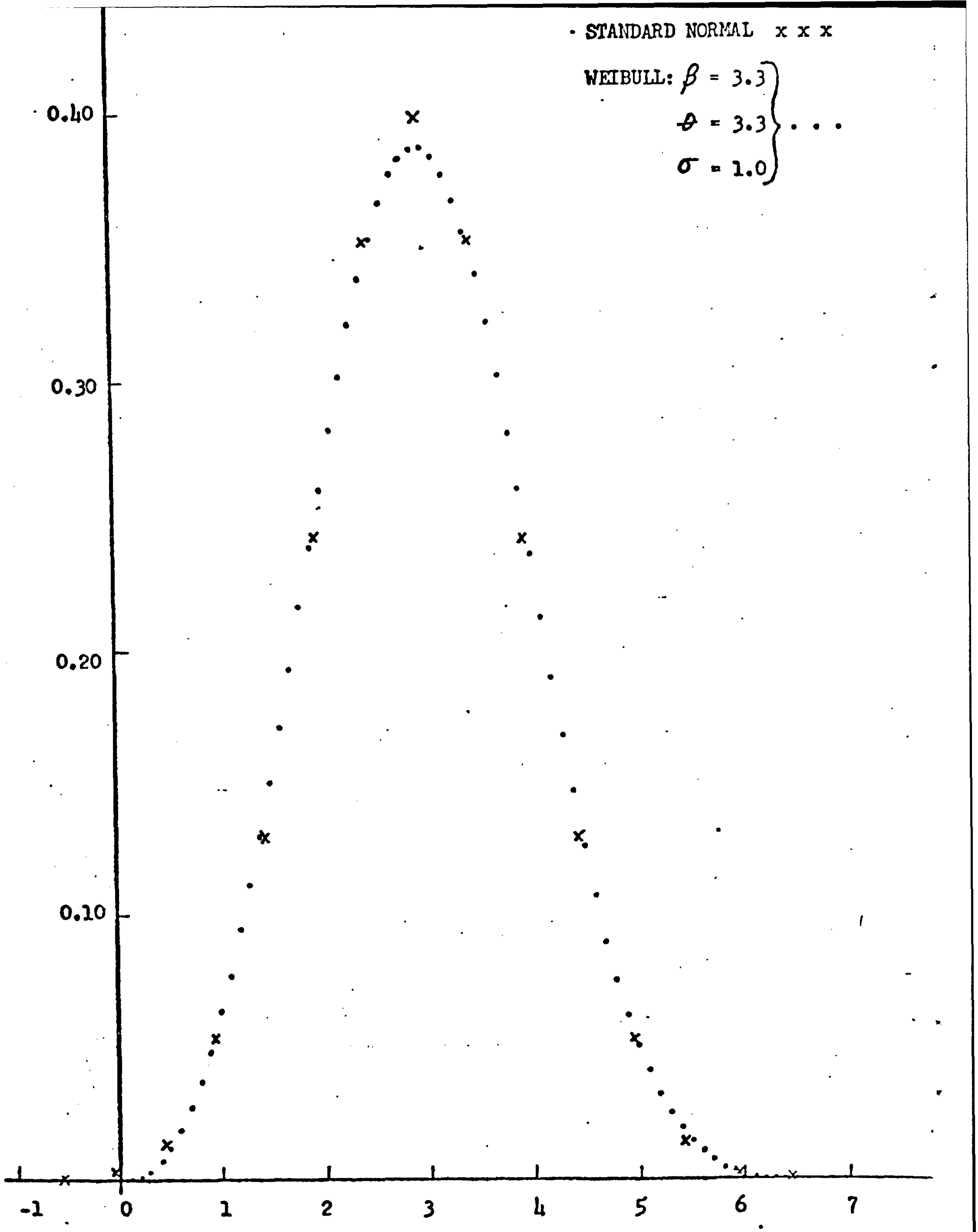
> "the exponential distribution is applicable as a model for failure times only if the failure rate is constant over time. In reality, failure rates which change with time are sometimes encountered. The normal distribution is a realistic model only if an increasing failure rate is encountered. The Weibull distribution is continuous and can account for a decreasing failure rate." (4)

To further illustrate the usefulness of the Weibull distribution, reference is made to Figure 12. It illustrates the typical life history of a population of units of a complex product. The initial or "de-bugging" phase is caused by the short life of marginal units; this phase is characterized by a high but decreasing failure rate. This phase of the life history can be handled with the Weibull distribution using $\beta < 1$. The second phase is characterized by a low and relatively constant failure rate which lasts till the units begin to wear out. The failure rate



·STANDARD NORMAL x x x
WEIBULL: β = 3.0
θ = 3.0
σ = 1.0
. . .
0.40
0.30
0.20
0.10
-1
0
1
2
3
4
5
6
7

FIGURE 4



STANDARD NORMAL x x x
WEIBULL: β = 3.3
θ = 3.3
σ = 1.0
0.40
0.30
0.20
0.10
-1
0
1
2
3
4
5
6
7

FIGURE 5

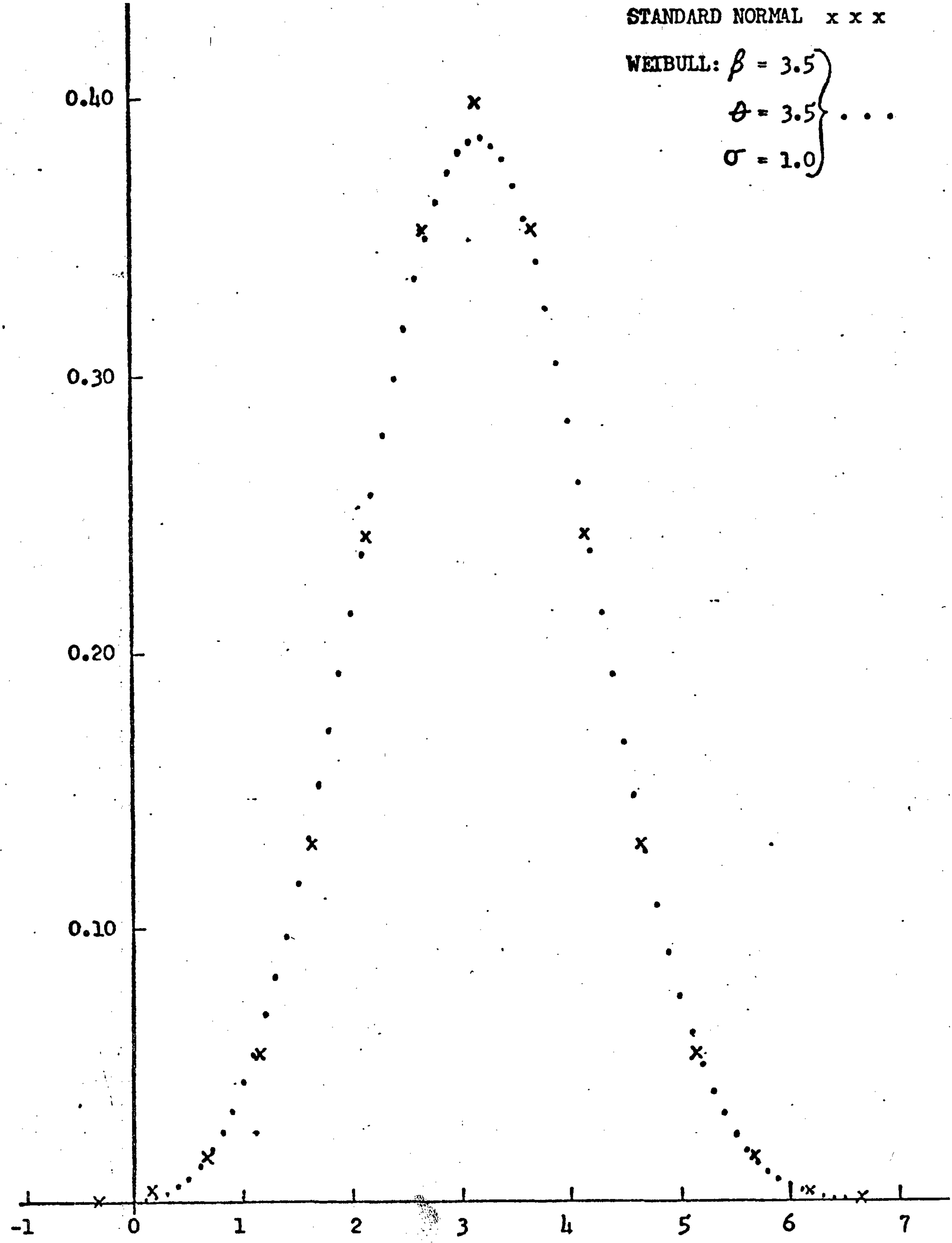

STANDARD NORMAL x x x
WEIBULL: $\beta = 3.5$, $\theta = 3.5$, $\sigma = 1.0$ . . .
0.40
0.30
0.20
0.10
-1
0
1
2
3
4
5
6
7

FIGURE 6

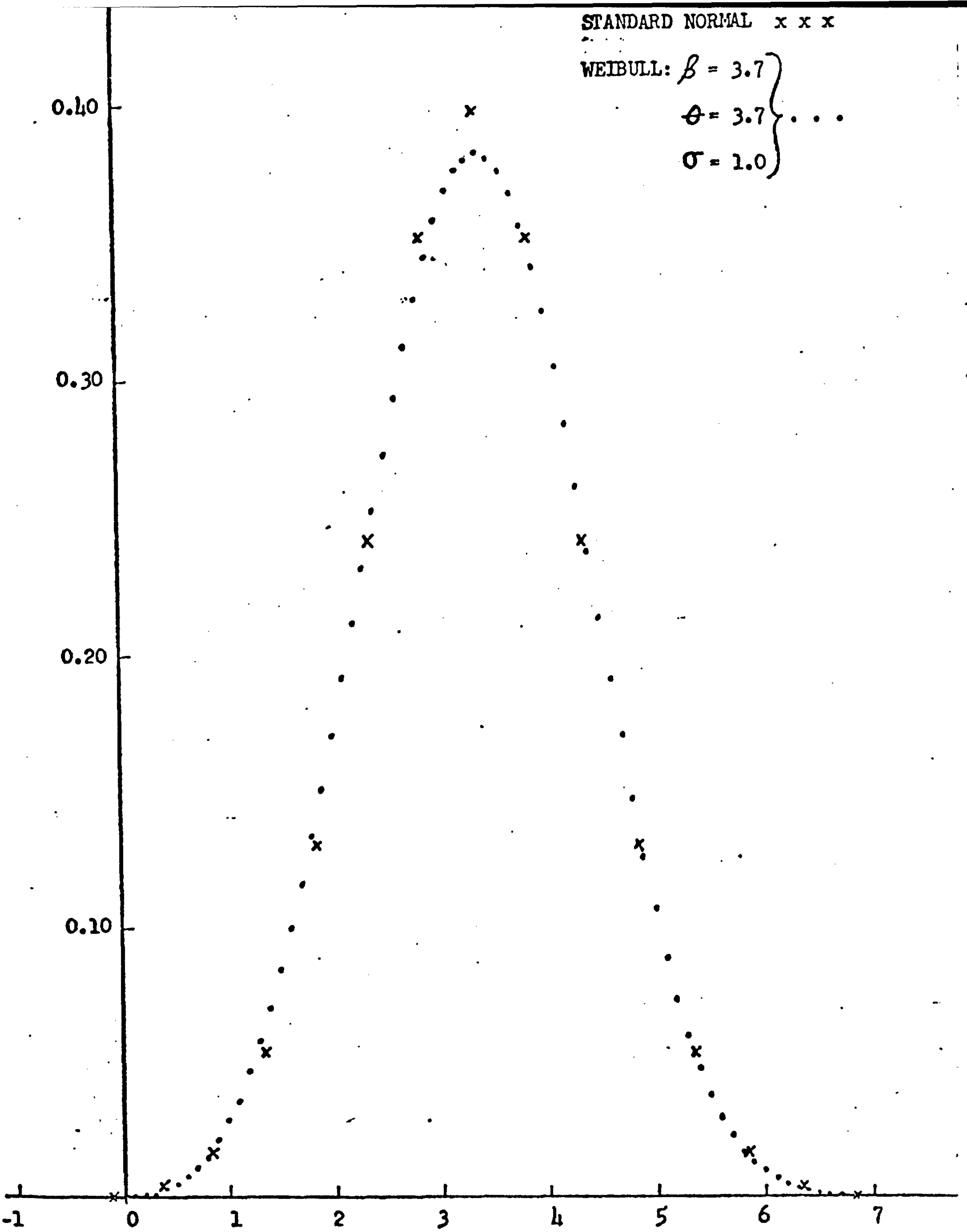

STANDARD NORMAL x x x
WEIBULL: β = 3.7
θ = 3.7
σ = 1.0
0.40
0.30
0.20
0.10
-1
0
1
2
3
4
5
6
7

FIGURE 7

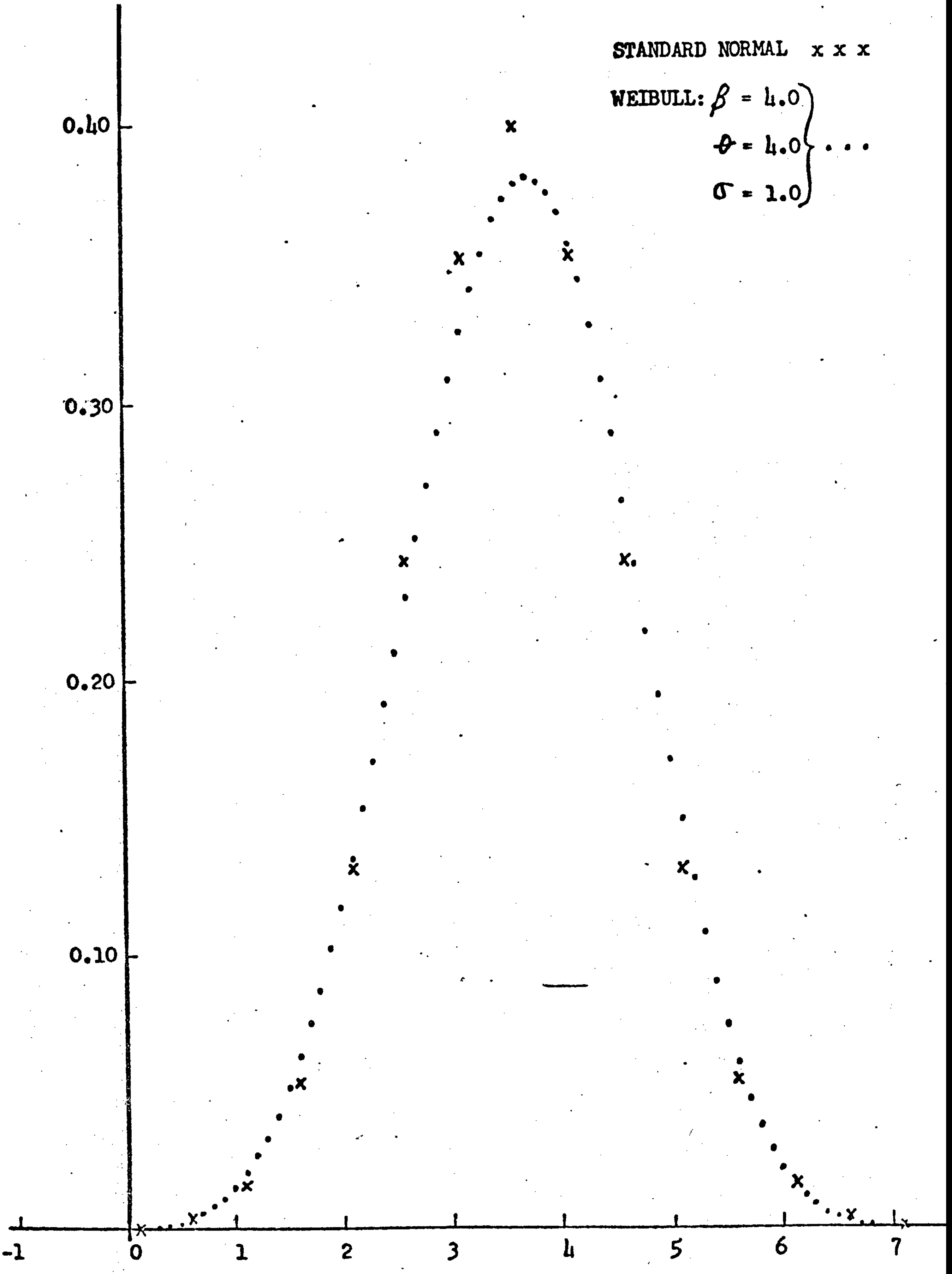

STANDARD NORMAL x x x
WEIBULL: β = 4.0
θ = 4.0
σ = 1.0
. . .
0.40
0.30
0.20
0.10
-1
0
1
2
3
4
5
6
7

FIGURE 8

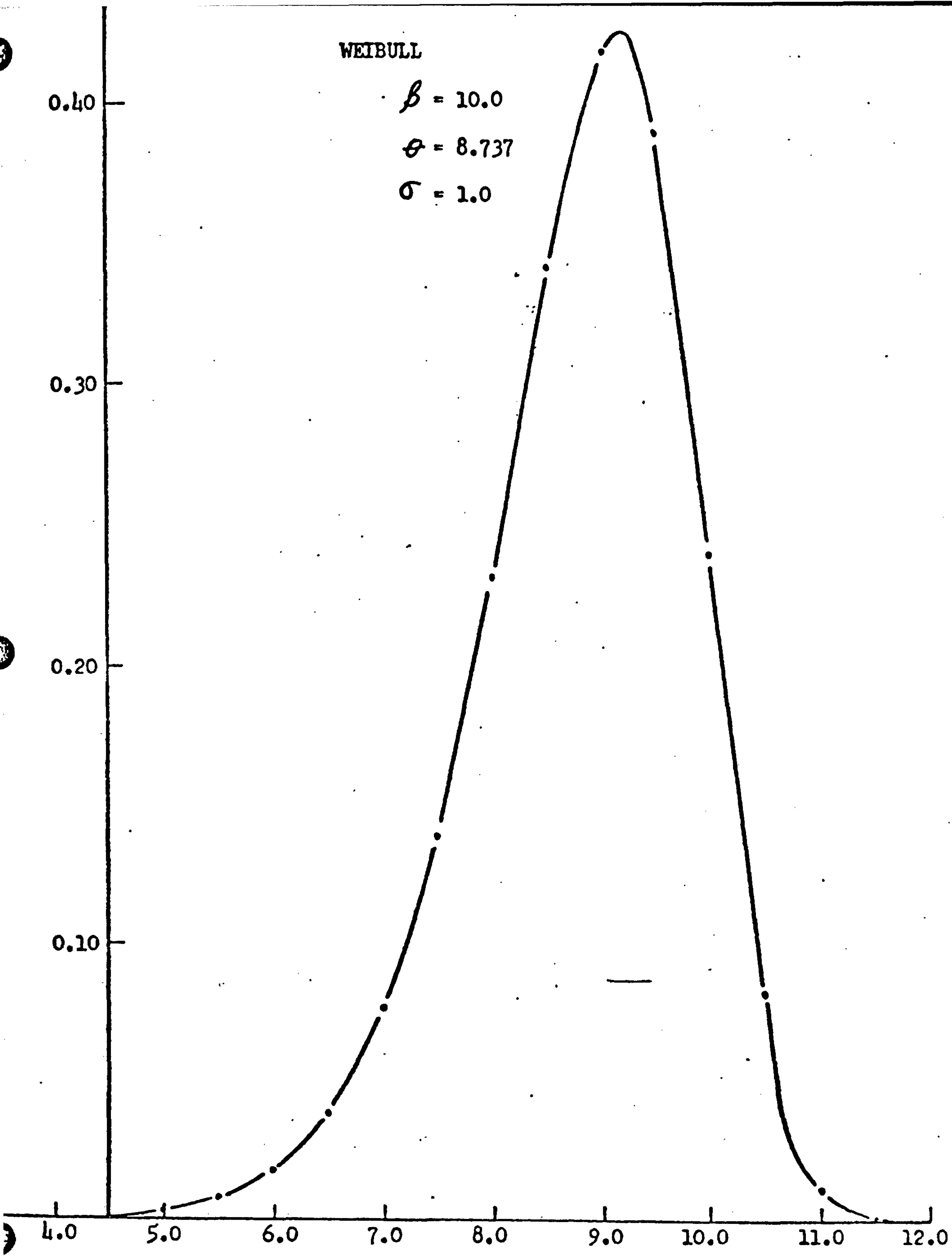

WEIBULL
β = 10.0
θ = 8.737
σ = 1.0
0.40
0.30
0.20
0.10
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0

FIGURE 9

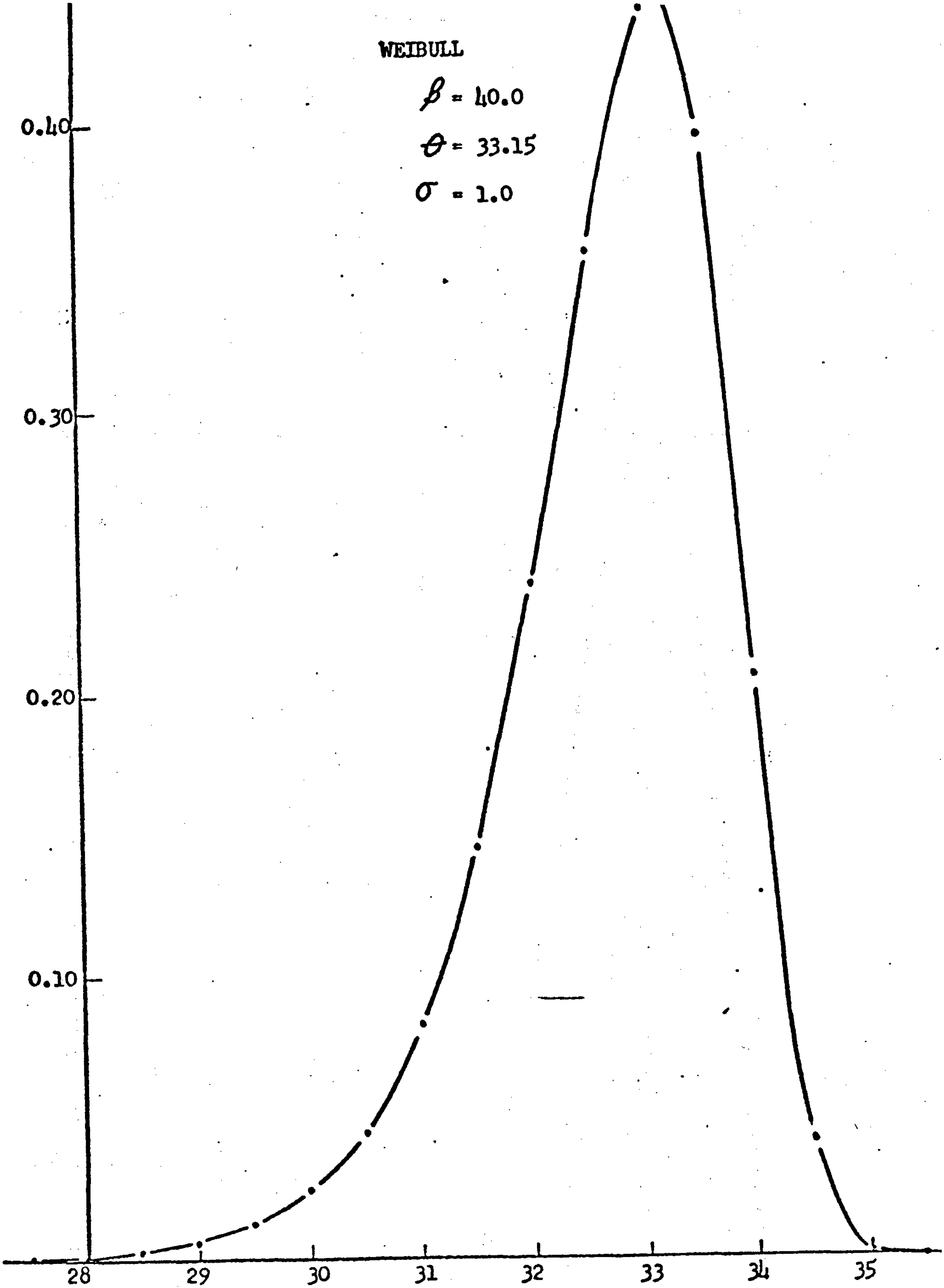


FIGURE 10

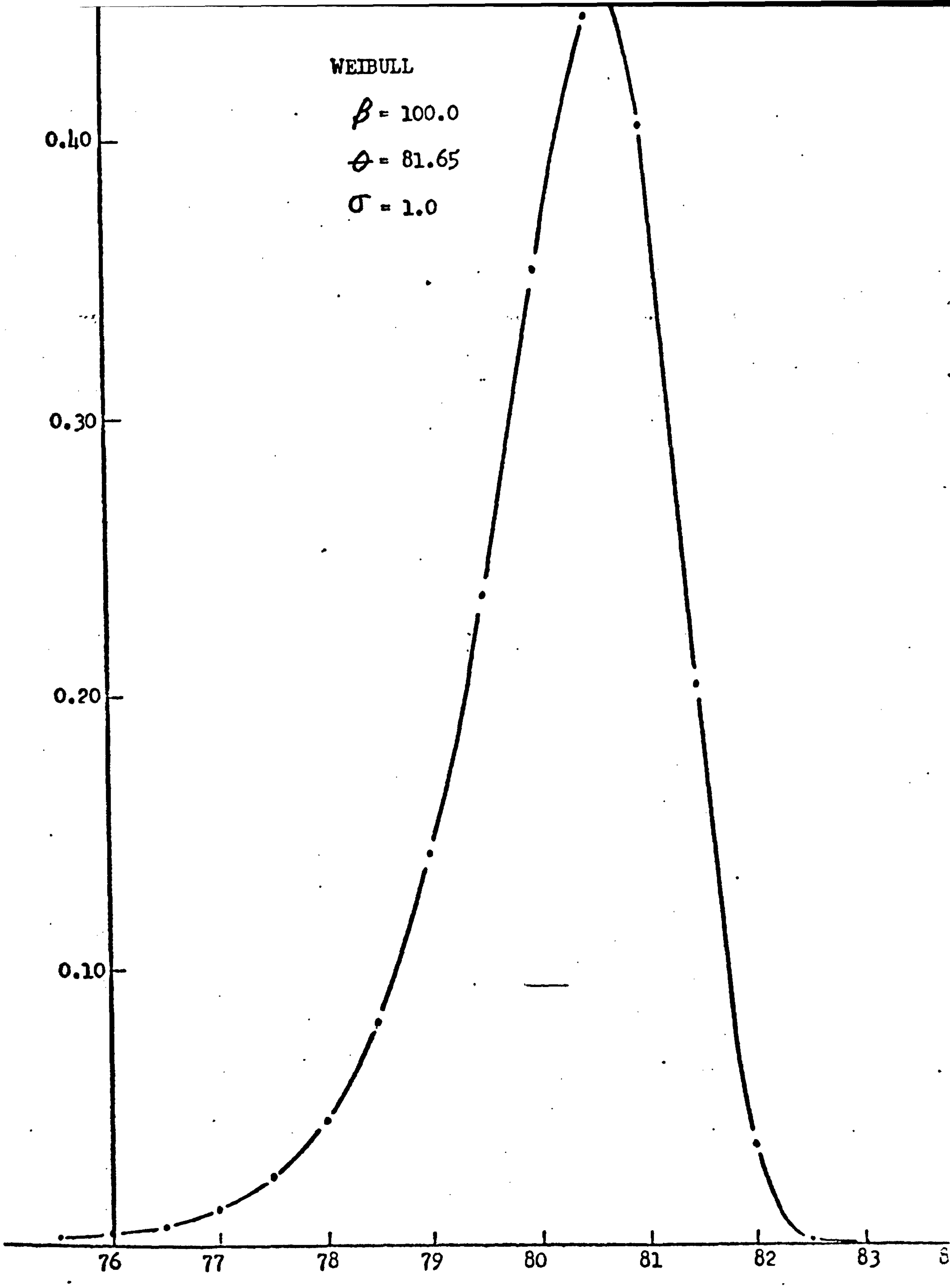


FIGURE 11

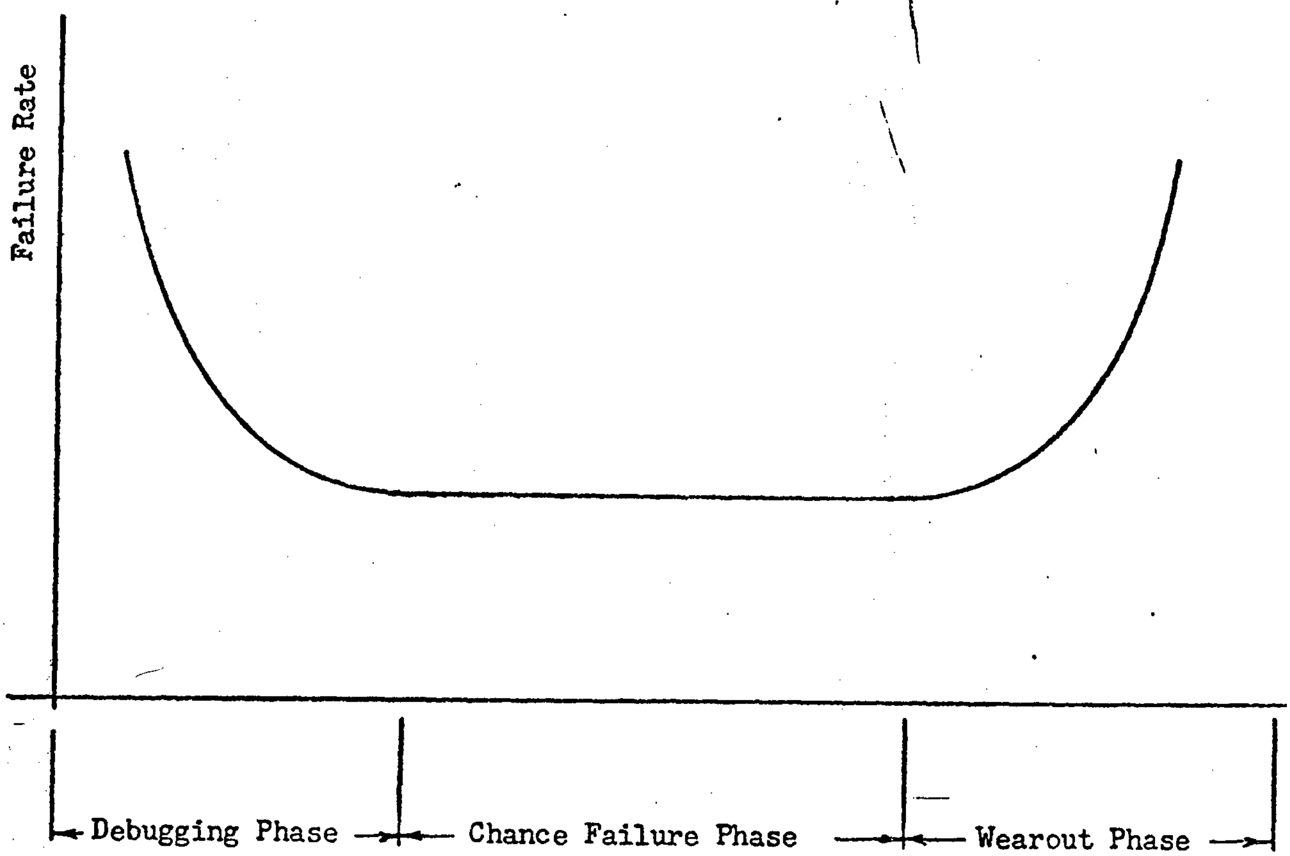


FIGURE 12. Typical life history of a population of units of a complex product (not to scale regarding phase lengths).

during this period is due to chance failures. This portion of the life history can be handled with the Weibull distribution using $\beta = 1$. The last phase is the wear out phase and is characterized by an increasing failure rate. This portion of the life history can be handled with the Weibull distribution using $\beta > 1$. (5) It is seen then that the Weibull distribution can be employed in failure analysis over the entire life history of the product, and is consequently of considerable value in failure analysis.

The computer was employed in this study to simulate:

1. Weibull distributed failures via Monte-Carlo techniques, and

2. Use of Weibull probability paper for estimating the values of β and θ.

A description of the Monte-Carlo technique was given earlier in this report as part of the section on Test Procedures. The following is a description of the use of Weibull probability paper for estimating β and θ.

This method for estimating β and θ employs graph paper called Weibull probability paper. The graph paper has its axes so graduated that when Weibull distributed failures are plotted, a straight line plot will result. An example of Weibull probability paper is shown in Figure 13. Failure times, t, are plotted along the horizontal axis. The vertical axis represents the cumulative Weibull probability, F(t) (see equation (1)).

Actual life testing consists of selecting n test samples from a large but unspecified sized population, and running them to failure. The only data that is obtained from such a test is the various values of time to failure, t_j, where $j = 1$ for the first failure, $j = 2$ for the second failure, etc., up to $j = n$ for the last failure. These values of failure times are plotted along the horizontal axis. The corresponding values of $F(t_j)$ are obtained from an estimator and are plotted along the vertical axis. The most commonly used estimator for this purpose is called the "mean ranks". Algebraically, it is expressed as $j/(n + 1)$. Using the symbol "$\wedge$" above a quantity to indicate "estimated value of", we have

$$\widehat{F(t_j)} = j/(n + 1)$$

for the mean ranks estimation. The physical meaning of "mean ranks" is that if the same failure analysis test were conducted many times, the

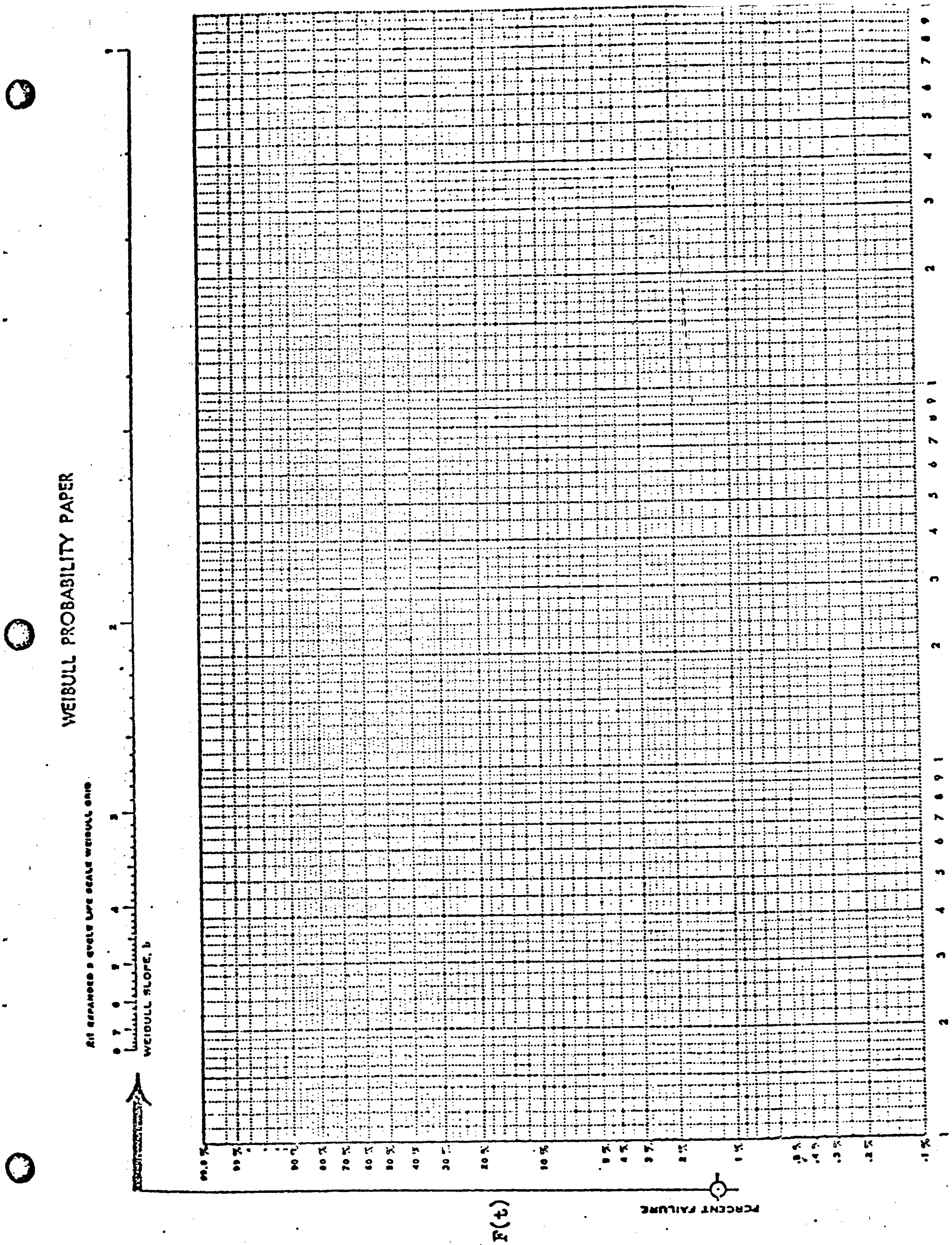

WEIBULL PROBABILITY PAPER
WEIBULL SLOPE, b
PERCENT FAILURE
F(t)

FIGURE 13

mean value for a particular $F(t_j)$ would be $j/(n+1)$. That is:

the mean value for $F(t_1)$ would be $1/(n + 1)$,
the mean value for $F(t_2)$ would be $2/(n + 1)$,

.
.
.

the mean value for $F(t_n)$ would be $n/(n + 1)$.

The derivation of the expression $j/(n + 1)$ for the mean ranks is shown by L. G. Johnson (6) and L. R. Lamberson (7). Messrs Kao and Goode (8), (9), (10) have done much work with the Weibull distribution; much of their studies make use of mean ranks. The United States Army (4) prescribes the use of mean ranks in connection with life testing using the Weibull distribution.

Another estimator used for this purpose is called the "median ranks". The exact values of the median ranks are obtained from tables, however, an approximating equation for the median ranks is given by E. J. Gumbell (11) as $(j - .3)/(n + .4)$. Using it we have

$$\widehat{F(t_j)} = (j - .3)/(n + .4)$$

for the median ranks estimation. The physical meaning of "median ranks" is that if the same failure analysis experiment were conducted many times, the median value for a particular $F(t_j)$ would be $(j - .3)/(n + .4)$. That is

the mean value for $F(t_1)$ would be $(1 - .3)/(n + .4)$,
the mean value for $F(t_2)$ would be $(2 - .3)/(n + .4)$,

.
.
.

the mean value for $F(t_n)$ would be $(n - .3)/(n + .4)$.

Mr. L. G. Johnson has written many papers (6), (12) and books (13), (14) dealing with analysis using the Weibull probability distribution. He favors use of median ranks when hand drawing the straight line on Weibull probability paper. He writes (15):

"To draw the line very near the lower extreme values when mean ranks are used would lead to a slope which is too small, because of the high probability of lower extreme values falling considerably to the left in such a case. On the other hand, it has been found that if median ranks are used in plotting, then the danger of under estimating the slope is eliminated because, in this case, a point is just as liable to fall to the right as to the left of the maximum likelihood line. In other words, we can very quickly arrive at an estimate of the population parameters by drawing a line by sight which takes the general direction of the array of points and which splits the array 50-50. For this reason median ranks are preferred to mean ranks."

It should be noted that regardless of which estimator (mean ranks, or median ranks) is used, $0 \leq F(t_j) \leq 1$. In terms of percentages, $F(t_j)$ lies between 0% and 100%. In the life test experiment described above, $F(t_j)$ represents percentage-wise, the number of samples out of the entire population of samples that will have a lifetime less than or equal to the lifetime of the j^{th} test sample to fail.

If it is not possible to draw a straight line through the points plotted on Weibull probability paper, then either the failures are not Weibull distributed or the minimum life parameter, α, is not zero. If the failures are not Weibull distributed, then Weibull probability paper cannot be used. If α is not zero, the data can be adjusted via a transformation of the failure time axis by an amount α; then the adjusted points can be connected by a straight line.

Estimates of the parameters β and θ are obtained directly from the straight line plot. As will be shown below, the slope of the straight line is β, and the intercept of the straight line on the vertical axis is $-\beta \ln \theta$, from which one can obtain the value of θ. In general practice, however, it is more common to obtain the value of θ merely by evaluating

$$t/\theta = 1$$

where t is the value of failure time on the Weibull plot corresponding to $F(t) = 0.632$. The value 0.632 is derived from the fact that for any value of β, the value of $F(t)$ obtained from equation (1) when $t/\theta = 1$ is always 0.632. In this study, the latter method for determining θ was not used. It was found to be equally expeditious in the computer program to evaluate θ by equating the intercept on the vertical axis to $-\beta \ln \theta$.

To show that the slope of the straight line plot on Weibull probability paper is β, we start with equation (1):

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]$$

and obtain the following:

$$\frac{1}{1 - F(t)} = e^{\left(\frac{t}{\theta}\right)^{\beta}}$$

$$\ln \ln \left(\frac{1}{1 - F(t)}\right) = \beta \ln t - \beta \ln \theta \qquad (4)$$

Equation (4) is seen to be of the form $Y = \beta X + B$

when we let $Y = \ln \ln \left(\frac{1}{1 - F(t)}\right)$ (5)

$X = \ln t$ (6)

$B = -\beta \ln \theta$, (7)

in which case β is the slope of a straight line plot on a coordinate system where the horizontal axis is graduated according to $X = \ln t$ and the vertical axis is graduated according to $Y = \ln \ln (1/(1 - F(t)))$. Weibull probability paper has its axes so graduated, consequently β is the slope of a straight line plot on Weibull probability paper of $F(t)$ vs. failure time, t. Note that the intercept of the straight line plot is B and that it equals $-\beta \ln \theta$. It is seen then that θ can be calculated once values for β and B are obtained.

It should be noted that because estimators such as mean ranks or median ranks are used in this method as estimations for $F(t)$, the resulting calculated values of β and θ are estimates of β and θ and should be labeled $\hat{\beta}$ and $\hat{\theta}$ to indicate that they are estimated values.

Sometimes in life testing, the test is cut short or stopped prior to having all of the test samples fail. For example, if 40% of the test samples have not yet failed when the test is terminated, the test is termed a 40% suspended data test. If 60% of the test samples have not failed by the time the test is terminated, the test is termed a 60% suspended data test. Suspended data tests are intuitively expected to be less accurate than non-suspended data tests for estimating the Weibull parameters β and θ because they result in fewer points on the Weibull probability paper through which to draw the best fit straight line.

APPENDIX II

COMPUTER PROGRAM

```
WEIBUL,CM60000,T500.
COMMENT.(TAA-102,00000D),CATALANO
FTN.
LGO.
0000000000000000000000
      PROGRAM PR685S(INPUT, OUTPUT)
C
C     SIMULATION OF WEIBULL FAILURES
C     COMPARISON OF MEDIAN RANKS AND MEAN RANKS
C     AS WEIBULL RANKING APPROXIMATORS
C     COMPARISON OF APPROXIMATIONS WHEN USING 40%,60% AND 0% SUSPENSION
C
C              ***MAIN PROGRAM***
C
      DIMENSION Y(160),T(160),RMED(160),RMEN(160),
     SBETMED(20,3),FTMED(20,3),THMED(20,3),
     SBETMEN(20,3),FTMEN(20,3),THMEN(20,3),
     SX(160),XRMED(160),XRMEN(160)
      REAL MDBMED(3),MDBMEN(3),MDTMED(3),MDTMEN(3)
C
C                  PRINT HEADINGS
C
      PRINT14
   14 FORMAT(≠1≠,1X,17H****VARIABLES****,17X,85H***********RESULTS---STA
     SNDARD ERROR (FOR 20 MONTE-CARLO SIMULATION TRIALS)**********,//,
     S38X,24H******MEDIAN RANKS******,32X,24H*******MEAN RANKS*******,
     S//,39X,21HPERCENTAGE SUSPENSION,35X,21HPERCENTAGE SUSPENSION,
     S//,1X,6HSAMPLE,22X,3H60%,16X,3H40%,15X,4H 0% ,15X,3H60%,16X,
     S3H40%,15X,4H 0% ,/,1X,4HSIZE,2X,4HBETA,3X,5HTHETA,5X,4HBETA,5X,
     S5HTHETA,5X,4HBETA,5X,5HTHETA,5X,4HBETA,3X,5HTHETA,7X,4HBETA,5X,
     S5HTHETA,5X,4HBETA,5X,5HTHETA,5X,4HBETA,3X,5HTHETA)
C
C                  ASSIGNMENT OF PARAMETER VALUES (THETA, BETA, N)
C
      DO 12 I2=1,6
      PRINT20
   20 FORMAT(/)
      THETA=5000.0*2**(I2-1)
      DO 10 L=1,5
      PRINT18
   18 FORMAT(/)
      BETA=(2**L)/4.0
      DO 8 K=1,6
      N=5*2**(K-1)
      DO 6 M=1,20
```

```
C
C                       GENERATE A SET OF N RANDOM NUMBERS
C
      DO 3 I=1,N
   30 Y(I)=RANF(0)
      IF(Y(I).LT.0.0)GO TO 30
    3 CONTINUE
C
C                       RANK ORDER THE SET OF RANDOM NUMBERS
C
      DO 5 I=1,N
      DO 7 J=I,N
      IF(Y(I)-Y(J))7,7,9
    9 S=Y(I)
      Y(I)=Y(J)
      Y(J)=S
    7 CONTINUE
    5 CONTINUE
C
C                       GENERATE N MONTE-CARLO FAILURE TIMES
C                                N MEDIAN RANKS
C                                N MEAN RANKS
C
      DO 11 I=1,N
      T(I)=(THETA)*(-ALOG(1.0-Y(I)))**(1.0/BETA)
      RMED(I)=(I-0.3)/(N+0.4)
      RMEN(I)=I/(N+1.0)
   11 CONTINUE
C
C                       GENERATE N TRANSFORMED AXES VALUES OF
C                                FAILURE TIMES:X
C                                MEDIAN RANKS: XRMED
C                                MEAN RANKS:   XRMEN
C
      DO 1 I=1,N
      X(I)=ALOG(T(I))
      XRMED(I)=ALOG(ALOG(1.0/(1.0-RMED(I))))
    1 XRMEN(I)=ALOG(ALOG(1.0/(1.0-RMEN(I))))
```

```
C
C                    PERFORM LEAST SQUARES FITTING FOR MEDIAN RANKS
C
      DO 40 I=1,3
      N=5*2**(K-1)
      N=IFIX((0.2+0.2*2**(I-1))*N)
      CALL LSFIT(N,X,XRMED,A,B,C,D,E,F)
      BETMED(M,I)=(C*E-B*F)/(A*E-B*D)
      FTMED(M,I)=(A*F-C*D)/(A*E-B*D)
      THMED(M,I)=EXP(-(FTMED(M,I))/(BETMED(M,I)))

C
C                    PERFORM-LEAST-SQUARES-FITTING-FOR-MEAN-RANKS
C
      CALL LSFIT(N,X,XRMEN,A,B,C,D,E,F)
      BETMEN(M,I)=(C*E-B*F)/(A*E-B*D)
      FTMEN(M,I)=(A*F-C*D)/(A*E-B*D)
      THMEN(M,I)=EXP(-(FTMEN(M,I))/(BETMEN(M,I)))
   40 CONTINUE
    6 CONTINUE
C
C                    PERFORM STANDARD ERROR CALCULATIONS
C
      DO 50 I=1,3
      CALL MDEV(I,BETA,BETMED,MDBMED)
      CALL MDEV(I,BETA,BETMEN,MDBMEN)
      CALL MDEV(I,THETA,THMED,MDTMED)
      CALL MDEV(I,THETA,THMEN,MDTMEN)
   50 CONTINUE
      PRINT16,N,BETA,THETA,MDBMED(1),MDTMED(1),MDBMED(2),MDTMED(2),MDBME
     SD(3),MDTMED(3),MDBMEN(1),MDTMEN(1),MDBMEN(2),MDTMEN(2),MDBMEN(3),
     SMDTMEN(3)
   16 FORMAT(1X,I3,3X,F3.1,2X,F8.1,3X,F6.3,1X,F10.1,2X,F6.3,1X,F10.1,
     S2X,F5.3,1X,F9.1,3X,F6.3,1X,F10.1,2X,F6.3,1X,F10.1,2X,F5.3,2X,F9.1)
    8 CONTINUE
   10 CONTINUE
   12 CONTINUE
      STOP
      END
C
```

```
C
C                    ***SUBROUTINES***
C
C
C                         SUBROUTINE FOR LEAST SQUARES FITTING
C
      SUBROUTINE LSFIT(N,T,R,A,B,C,D,E,F)
      REAL T(160),R(160)
      SUMX=0.0
      DO 1 J=1,N
    1 SUMX=SUMX+T(J)
      A=SUMX
      B=N/1.0
      SUMY=0.0
      DO 2 J=1,N
    2 SUMY=SUMY+R(J)
      C=SUMY
      SUMX2=0.0
      DO 3 J=1,N
    3 SUMX2=SUMX2+T(J)**2
      D=SUMX2
      E=A
      SUMXY=0.0
      DO 4 J=1,N
    4 SUMXY=SUMXY+T(J)*R(J)
      F=SUMXY
      RETURN
      END
C
C     SUBROUTINE FOR CALCULATING STANDARD ERROR
C
      SUBROUTINE MDEV(I,KNOWN,EST,MENDEV)
      REAL KNOWN,MENDEV(3),EST(20,3),DEV2(20,3)
      DO 1 M=1,20
    1 DEV2(M,I)=(KNOWN-EST(M,I))**2
      SUMDEV=0.0
      DO 2 M=1,20
    2 SUMDEV=SUMDEV+DEV2(M,I)
      DUMMY=SUMDEV/20.0
      MENDEV(I)=SQRT(DUMMY)
      RETURN
      END
00000000000000000000
```

APPENDIX III

CHARTED PRINTOUT OF RESULTS

CHART NO. 1

PRINTOUT OF STANDARD ERROR WHEN USING MEAN RANKS

STANDARD ERROR

*******MEAN RANKS*******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	5000.0	*7.469	4793735.8	*7.469	4793735.8	.234	8128.7
10	.5	5000.0	.305	195047.7	.232	46925.9	.130	4495.6
20	.5	5000.0	.241	58775.4	.175	14709.9	.111	2936.5
40	.5	5000.0	.208	95934.9	.150	18468.0	.088	2794.6
80	.5	5000.0	.136	13663.8	.112	4114.4	.059	984.3
160	.5	5000.0	.089	6871.7	.070	3040.2	.046	968.2
5	1.0	5000.0	3.974	28086.3	3.974	28086.3	.877	2295.1
10	1.0	5000.0	.619	8929.7	.483	6144.2	.285	1804.0
20	1.0	5000.0	.363	13606.8	.307	4681.5	.210	1022.3
40	1.0	5000.0	.317	2433.2	.223	1435.3	.163	817.3
80	1.0	5000.0	.186	2890.1	.143	1370.8	.089	577.4
160	1.0	5000.0	.158	2541.2	.134	1139.6	.092	436.7
5	2.0	5000.0	5.793	11001.4	5.793	11001.4	1.002	1385.1
10	2.0	5000.0	2.967	6762.9	1.220	4010.4	.513	738.3
20	2.0	5000.0	.814	3033.3	.661	1637.8	.460	589.1
40	2.0	5000.0	.609	2160.4	.555	1164.1	.389	474.2
80	2.0	5000.0	.369	954.3	.335	602.0	.240	247.3
160	2.0	5000.0	.416	1156.9	.344	536.3	.240	192.5
5	4.0	5000.0	21.403	1701.4	21.403	1701.4	2.766	591.3
10	4.0	5000.0	2.641	2330.0	2.443	1593.2	1.670	414.7
20	4.0	5000.0	1.890	1661.8	1.467	929.9	1.003	272.1
40	4.0	5000.0	1.203	607.1	.983	348.2	.797	205.9
80	4.0	5000.0	1.064	623.6	.952	360.4	.594	158.3
160	4.0	5000.0	.624	300.8	.513	187.8	.318	93.3
5	8.0	5000.0	24.652	811.8	24.652	811.8	2.852	325.5
10	8.0	5000.0	5.030	1117.9	4.452	748.6	2.559	156.8
20	8.0	5000.0	2.920	658.1	2.356	421.2	1.728	206.9
40	8.0	5000.0	2.678	425.4	2.556	267.9	1.497	121.0
80	8.0	5000.0	1.815	291.8	1.518	165.5	1.016	61.4
160	8.0	5000.0	1.459	234.3	1.101	133.7	.647	53.2

STANDARD ERROR

*******MEAN RANKS*******

****VARIABLES****			PERCENTAGE SUSPENSION					
			60%		40%		0%	
SAMPLE SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	10000.0	2.629	*4440491.8	2.629	*4440491.8	.354	21169.2
10	.5	10000.0	.819	*0765708.3	.458	22425170.1	.195	12681.0
20	.5	10000.0	.244	460722.5	.179	85459.1	.116	8162.6
40	.5	10000.0	.146	37054.1	.119	12070.1	.072	3669.2
80	.5	10000.0	.120	12775.7	.095	7090.5	.056	2844.1
160	.5	10000.0	.075	9613.8	.061	3814.8	.044	1741.9
5	1.0	10000.0	4.778	38094.2	4.778	38094.2	.381	5331.6
10	1.0	10000.0	.861	28821.0	.687	16624.8	.302	4504.4
20	1.0	10000.0	.352	47863.2	.309	16022.6	.202	2846.9
40	1.0	10000.0	.310	17122.1	.277	7238.7	.190	2806.7
80	1.0	10000.0	.187	5040.9	.157	2677.9	.103	1160.9
160	1.0	10000.0	.173	4572.9	.133	2402.2	.075	926.4
5	2.0	10000.0	3.183	9223.4	3.183	9223.4	.982	2193.7
10	2.0	10000.0	.876	8515.8	.826	5293.5	.592	1597.1
20	2.0	10000.0	.613	4009.6	.572	2859.0	.437	1241.3
40	2.0	10000.0	.506	2498.5	.362	1292.1	.269	825.3
80	2.0	10000.0	.361	3702.0	.287	1814.6	.205	797.9
160	2.0	10000.0	.314	2011.4	.251	1101.7	.158	511.5
5	4.0	10000.0	17.023	3178.2	17.023	3178.2	3.211	1334.5
10	4.0	10000.0	2.460	4279.8	2.176	2915.6	1.470	844.1
20	4.0	10000.0	2.853	3229.4	1.805	1918.4	1.016	627.1
40	4.0	10000.0	1.250	2449.4	1.108	1430.1	.783	438.6
80	4.0	10000.0	.611	674.2	.508	378.3	.382	262.0
160	4.0	10000.0	.620	666.2	.534	447.6	.316	226.6
5	8.0	10000.0	19.457	1994.3	19.457	1994.3	2.932	625.7
10	8.0	10000.0	3.908	2276.4	3.386	1568.3	2.167	482.4
20	8.0	10000.0	3.690	1576.6	3.038	916.2	2.137	250.6
40	8.0	10000.0	2.437	917.3	2.248	548.2	1.659	214.5
80	8.0	10000.0	1.501	528.8	1.193	291.1	.758	132.0
160	8.0	10000.0	1.056	305.1	.879	197.1	.603	109.5

STANDARD ERROR

*******MEAN RANKS*******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	20000.0	4.288	*3955750.0	4.288	*3955750.0	.272	34192.6
10	.5	20000.0	.323	15059818.5	.325	1507381.4	.134	13472.2
20	.5	20000.0	.207	1186650.0	.171	139793.8	.127	11376.9
40	.5	20000.0	.141	82708.1	.114	32487.1	.075	11377.0
80	.5	20000.0	.134	116422.5	.109	25640.8	.073	5778.6
160	.5	20000.0	.097	31573.3	.074	11886.5	.048	3371.6
5	1.0	20000.0	7.635	35196.9	7.635	35196.9	.429	9123.4
10	1.0	20000.0	.765	99045.9	.729	50296.8	.327	10949.3
20	1.0	20000.0	.357	26047.9	.340	13189.7	.241	6365.5
40	1.0	20000.0	.302	28543.5	.257	12235.7	.187	4428.8
80	1.0	20000.0	.190	12713.7	.162	5945.1	.122	2519.8
160	1.0	20000.0	.150	5980.9	.117	3398.3	.077	1525.2
5	2.0	20000.0	6.350	72237.7	6.350	72237.7	1.092	6098.1
10	2.0	20000.0	1.259	16059.0	1.188	11344.2	.666	3627.[illegible]
20	2.0	20000.0	.690	11968.2	.604	7079.0	.453	2438.9
40	2.0	20000.0	.536	7279.0	.479	3365.6	.331	1632.3
80	2.0	20000.0	.360	4132.0	.311	2484.7	.219	1254.2
160	2.0	20000.0	.260	2954.9	.199	1907.7	.127	1006.5
5	4.0	20000.0	8.753	8710.7	8.753	8710.7	1.441	2431.1
10	4.0	20000.0	1.984	13555.8	1.857	8449.5	1.413	1952.9
20	4.0	20000.0	2.124	5204.6	1.698	3100.5	.978	1368.6
40	4.0	20000.0	1.233	6249.9	1.042	3053.5	.717	809.4
80	4.0	20000.0	.967	3149.9	.811	1705.8	.583	519.2
160	4.0	20000.0	.604	1713.6	.439	964.1	.266	465.7
5	8.0	20000.0	*3.228	4002.7	*3.228	4002.7	4.047	1367.7
10	8.0	20000.0	5.115	4376.4	4.205	3168.6	2.463	880.2
20	8.0	20000.0	3.711	3430.0	3.389	2022.6	2.147	640.4
40	8.0	20000.0	2.181	1389.6	1.881	865.8	1.457	343.5
80	8.0	20000.0	1.981	1192.8	1.522	671.3	1.107	372.5
160	8.0	20000.0	1.209	681.0	1.011	340.0	.686	148.1

STANDARD ERROR

*******MEAN RANKS*******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	40000.0	.520	17076942.5	.520	17076942.5	.193	61540.3
10	.5	40000.0	.371	2951130.2	.315	736304.9	.156	39961.2
20	.5	40000.0	.302	849440.1	.239	184436.8	.092	25933.5
40	.5	40000.0	.172	838168.0	.153	132430.2	.096	11940.2
80	.5	40000.0	.095	39183.1	.073	20177.7	.049	10402.1
160	.5	40000.0	.076	43691.2	.059	21528.9	.042	7918.1
5	1.0	40000.0	2.765	174940.0	2.765	174940.0	.409	20646.0
10	1.0	40000.0	.628	106860.0	.525	43679.2	.310	9687.4
20	1.0	40000.0	.417	93283.4	.331	25918.8	.200	9648.7
40	1.0	40000.0	.287	151932.9	.245	38834.8	.180	9208.1
80	1.0	40000.0	.207	19031.1	.169	8365.9	.125	3929.9
160	1.0	40000.0	.167	13987.3	.137	7529.8	.086	3169.2
5	2.0	40000.0	9.450	35979.0	9.450	35979.0	1.359	10327.1
10	2.0	40000.0	1.119	29976.4	1.084	19052.3	.705	6845.9
20	2.0	40000.0	1.089	19473.4	.623	10778.1	.376	4506.2
40	2.0	40000.0	.633	9254.0	.588	5451.3	.328	2466.2
80	2.0	40000.0	.437	12158.4	.375	5487.9	.270	2568.2
160	2.0	40000.0	.349	5880.7	.299	3525.3	.201	1642.8
5	4.0	40000.0	7.333	57435.7	7.333	57435.7	1.926	4740.8
10	4.0	40000.0	1.766	20490.2	1.594	13809.9	1.161	3994.3
20	4.0	40000.0	1.518	9740.0	1.290	5244.7	.901	1565.4
40	4.0	40000.0	1.382	4956.6	1.254	3019.7	.693	1224.3
80	4.0	40000.0	.807	4488.3	.664	2715.1	.450	1176.3
160	4.0	40000.0	.596	3790.5	.470	2066.2	.306	751.6
5	8.0	40000.0	18.391	4704.8	18.391	4704.8	4.655	2051.0
10	8.0	40000.0	6.753	5783.2	4.497	4186.8	1.963	1743.6
20	8.0	40000.0	3.348	4030.3	2.525	2710.3	1.641	1254.1
40	8.0	40000.0	1.863	2334.3	1.720	1547.1	1.370	925.9
80	8.0	40000.0	1.317	1773.6	1.012	1136.9	.683	573.5
160	8.0	40000.0	1.202	1296.8	.906	745.3	.608	335.8

STANDARD ERROR

*******MEAN RANKS*******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	80000.0	5.854	*5670590.2	5.854	*5670590.2	.215	105767.3
10	.5	80000.0	.215	*4397384.6	.189	10982985.9	.145	56562.3
20	.5	80000.0	.197	1865397.8	.164	392070.4	.109	66366.3
40	.5	80000.0	.137	162364.9	.119	66025.1	.077	27295.9
80	.5	80000.0	.103	122425.3	.088	58116.0	.058	24988.5
160	.5	80000.0	.060	46840.0	.046	25320.1	.031	12586.6
5	1.0	80000.0	.632	374813.7	.632	374813.7	.710	37846.5
10	1.0	80000.0	.508	876369.9	.508	328428.0	.331	30734.8
20	1.0	80000.0	.344	152111.8	.286	72163.8	.226	25326.3
40	1.0	80000.0	.492	124317.4	.356	46804.2	.196	16137.5
80	1.0	80000.0	.250	53999.5	.199	27259.8	.125	13668.7
160	1.0	80000.0	.206	69729.3	.168	26738.4	.112	5966.7
5	2.0	80000.0	22.993	242498.1	22.993	242498.1	.791	19650.7
10	2.0	80000.0	1.063	75930.8	.839	45168.3	.470	14370.1
20	2.0	80000.0	.793	42328.8	.625	20258.4	.444	7881.6
40	2.0	80000.0	.713	37945.0	.615	19253.0	.401	6639.2
80	2.0	80000.0	.516	27514.6	.427	14068.9	.260	4822.7
160	2.0	80000.0	.416	18384.4	.350	9845.7	.219	2549.4
5	4.0	80000.0	39.543	50023.1	39.543	50023.1	1.897	9544.6
10	4.0	80000.0	1.721	46515.8	1.635	29226.7	1.154	7097.2
20	4.0	80000.0	1.349	13414.4	1.081	8519.5	.698	3379.8
40	4.0	80000.0	1.017	17277.6	.831	8843.8	.659	3106.1
80	4.0	80000.0	.883	8733.4	.745	5178.6	.494	2671.0
160	4.0	80000.0	.606	4942.7	.484	2712.7	.305	1705.9
5	8.0	80000.0	24.269	15204.1	24.269	15204.1	3.432	6457.8
10	8.0	80000.0	5.742	10399.2	5.659	6759.1	2.659	2944.4
20	8.0	80000.0	3.025	10930.6	2.428	6537.7	1.593	2107.5
40	8.0	80000.0	1.829	4131.7	1.630	2742.0	1.145	1488.9
80	8.0	80000.0	1.188	4110.8	1.009	2569.9	.695	1252.1
160	8.0	80000.0	1.572	3383.8	1.238	1512.4	.808	690.1

STANDARD ERROR

****VARIABLES****			*******MEAN RANKS******* PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	160000.0	2.601	*3324794.5	2.601	*3324794.5	.167	235484.4
10	.5	160000.0	.402	5504098.1	.277	1521504.1	.191	102813.7
20	.5	160000.0	.241	11543474.9	.195	1363642.8	.153	113046.8
40	.5	160000.0	.186	1943901.0	.148	433706.3	.093	80017.4
80	.5	160000.0	.128	769383.3	.086	166651.6	.049	43660.4
160	.5	160000.0	.062	86914.8	.055	44124.8	.044	27758.4
5	1.0	160000.0	7.489	542124.1	7.489	542124.1	.563	84344.2
10	1.0	160000.0	2.018	237742.5	1.127	131731.7	.317	51756.1
20	1.0	160000.0	.538	547686.4	.352	171842.4	.236	52792.8
40	1.0	160000.0	.263	124685.8	.196	55778.4	.129	17962.0
80	1.0	160000.0	.225	86972.5	.157	46806.0	.114	22425.5
160	1.0	160000.0	.136	57099.6	.102	28077.8	.066	13318.2
5	2.0	160000.0	54.392	76935.5	54.392	76935.5	.708	35594.5
10	2.0	160000.0	1.198	132753.7	1.108	77820.2	.580	25056.3
20	2.0	160000.0	1.014	105919.7	.879	52837.2	.461	24294.3
40	2.0	160000.0	.607	65914.0	.528	37286.9	.359	11505.2
80	2.0	160000.0	.391	33629.2	.319	18695.5	.213	7794.3
160	2.0	160000.0	.339	31807.9	.274	17383.0	.174	6621.3
5	4.0	160000.0	22.695	44114.6	22.695	44114.6	1.948	21968.5
10	4.0	160000.0	2.200	43813.5	2.126	33752.8	.915	11574.6
20	4.0	160000.0	2.371	31267.7	1.800	20013.9	1.017	9651.5
40	4.0	160000.0	.992	28297.3	.840	16263.3	.600	8020.1
80	4.0	160000.0	1.001	19599.4	.740	9278.1	.451	4480.6
160	4.0	160000.0	.557	11405.3	.394	6578.4	.281	3309.4
5	8.0	160000.0	44.571	26029.1	44.571	26029.1	5.050	8016.6
10	8.0	160000.0	7.859	24018.4	3.698	17887.0	2.430	5273.9
20	8.0	160000.0	3.499	22257.6	3.050	13341.0	2.032	4504.3
40	8.0	160000.0	2.654	12774.6	2.401	7037.1	1.481	3168.6
80	8.0	160000.0	1.796	9719.3	1.453	5937.2	.867	2915.7
160	8.0	160000.0	1.450	7063.0	1.201	4097.7	.812	1804.9

CHART NO. 2

PRINTOUT OF STANDARD ERROR WHEN USING MEDIAN RANKS

STANDARD ERROR

****VARIABLES****			******MEDIAN RANKS****** PERCENTAGE SUSPENSION					
			60%		40%		0%	
SAMPLE SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	5000.0	*9.794	1217960.9	*9.794	1217960.9	.264	6623.4
10	.5	5000.0	.381	87063.5	.276	27441.6	.121	4050.5
20	.5	5000.0	.261	32312.8	.175	10580.0	.100	2711.5
40	.5	5000.0	.224	59936.4	.154	14356.9	.081	2628.3
80	.5	5000.0	.142	9513.8	.113	3368.8	.055	950.1
160	.5	5000.0	.084	5561.6	.066	2678.8	.043	945.2
5	1.0	5000.0	5.086	16566.4	5.086	16566.4	1.054	2142.6
10	1.0	5000.0	.766	6345.9	.574	4752.7	.295	1711.6
20	1.0	5000.0	.366	9414.8	.301	3607.3	.188	1004.9
40	1.0	5000.0	.362	1925.3	.244	1252.6	.168	792.8
80	1.0	5000.0	.180	2335.1	.135	1200.4	.079	561.1
160	1.0	5000.0	.153	2151.3	.129	1019.4	.086	427.9
5	2.0	5000.0	7.447	7650.9	7.447	7650.9	1.177	1312.1
10	2.0	5000.0	3.501	5092.1	1.396	3210.3	.502	698.3
20	2.0	5000.0	.874	2382.1	.677	1350.3	.448	551.8
40	2.0	5000.0	.609	1746.1	.564	1025.6	.371	479.5
80	2.0	5000.0	.335	816.8	.314	547.8	.225	248.1
160	2.0	5000.0	.379	984.2	.315	468.2	.223	192.9
5	4.0	5000.0	27.236	1463.8	27.236	1463.8	3.383	559.9
10	4.0	5000.0	3.137	1883.2	2.832	1341.0	1.764	398.4
20	4.0	5000.0	2.067	1372.0	1.452	805.0	.853	273.1
40	4.0	5000.0	1.201	493.8	.925	301.5	.737	205.1
80	4.0	5000.0	1.066	526.8	.960	322.0	.586	157.0
160	4.0	5000.0	.603	263.2	.492	174.6	.302	94.8
5	8.0	5000.0	31.888	697.4	31.888	697.4	3.009	328.8
10	8.0	5000.0	5.962	909.5	5.032	627.0	2.344	152.9
20	8.0	5000.0	2.877	559.8	2.220	383.8	1.565	208.2
40	8.0	5000.0	2.654	365.1	2.528	238.3	1.364	118.2
80	8.0	5000.0	1.620	239.6	1.375	141.7	.952	60.3
160	8.0	5000.0	1.437	209.2	1.061	123.9	.600	52.2

STANDARD ERROR

******MEDIAN RANKS******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	10000.0	3.349	41443606.3	3.349	41443606.3	.428	17636.0
10	.5	10000.0	.991	*7996729.5	.535	6707458.6	.195	10664.6
20	.5	10000.0	.274	246833.4	.186	59160.2	.113	7500.9
40	.5	10000.0	.151	24710.1	.125	9380.7	.074	3438.0
80	.5	10000.0	.134	9750.2	.104	6029.3	.060	2732.7
160	.5	10000.0	.070	7595.5	.057	3282.7	.042	1701.3
5	1.0	10000.0	6.076	20448.8	6.076	20448.8	.341	4854.7
10	1.0	10000.0	1.054	19527.5	.815	12672.1	.338	4254.9
20	1.0	10000.0	.399	32092.3	.333	12473.7	.203	2551.1
40	1.0	10000.0	.291	12983.3	.266	6077.9	.178	2703.8
80	1.0	10000.0	.174	3988.7	.148	2292.5	.098	1123.5
160	1.0	10000.0	.175	3863.7	.136	2183.7	.077	907.2
5	2.0	10000.0	4.124	6953.5	4.124	6953.5	1.107	2185.3
10	2.0	10000.0	1.000	6192.8	.936	4157.2	.596	1512.6
20	2.0	10000.0	.606	3211.3	.532	2431.5	.380	1180.0
40	2.0	10000.0	.513	2009.9	.353	1117.6	.257	805.3
80	2.0	10000.0	.358	3154.7	.278	1642.0	.199	783.0
160	2.0	10000.0	.321	1763.2	.260	1001.0	.166	494.1
5	4.0	10000.0	21.750	2925.3	21.750	2925.3	3.853	1346.8
10	4.0	10000.0	2.752	3311.8	2.330	2392.0	1.394	850.5
20	4.0	10000.0	3.306	2639.7	2.009	1659.4	.974	605.7
40	4.0	10000.0	1.172	2059.7	1.025	1265.2	.690	415.9
80	4.0	10000.0	.557	592.3	.454	358.9	.334	265.4
160	4.0	10000.0	.614	610.4	.524	425.1	.300	227.4
5	8.0	10000.0	25.213	1763.6	25.213	1763.6	3.174	652.4
10	8.0	10000.0	4.478	1880.8	3.635	1331.3	2.109	470.8
20	8.0	10000.0	3.889	1293.7	3.017	771.7	1.970	239.0
40	8.0	10000.0	2.442	800.3	2.192	495.0	1.581	214.3
80	8.0	10000.0	1.253	436.9	1.022	250.8	.682	129.9
160	8.0	10000.0	.914	251.5	.775	173.7	.551	107.6

STANDARD ERROR

******MEDIAN RANKS******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	20000.0	5.409	33645872.1	5.409	33645872.1	.301	27622.1
10	.5	20000.0	.391	4430373.9	.382	645708.1	.117	12516.6
20	.5	20000.0	.204	412842.9	.160	79657.2	.113	10745.5
40	.5	20000.0	.137	57502.6	.103	26432.4	.064	10888.2
80	.5	20000.0	.134	80685.5	.106	20901.9	.069	5468.0
160	.5	20000.0	.101	25057.8	.076	10329.7	.049	3256.4
5	1.0	20000.0	9.660	22810.5	9.660	22810.5	.450	8415.4
10	1.0	20000.0	.920	66594.0	.856	37966.6	.332	10294.9
20	1.0	20000.0	.379	19596.2	.362	10800.1	.251	6131.3
40	1.0	20000.0	.276	21468.2	.239	10090.3	.175	4204.3
80	1.0	20000.0	.180	10491.9	.156	5260.4	.117	2479.6
160	1.0	20000.0	.154	5095.0	.119	3109.3	.075	1480.5
5	2.0	20000.0	8.154	48489.2	8.154	48489.2	1.318	5768.8
10	2.0	20000.0	1.536	11509.5	1.403	8789.5	.673	3516.9
20	2.0	20000.0	.676	9519.3	.574	5964.7	.443	2348.8
40	2.0	20000.0	.533	5709.0	.478	2820.3	.313	1607.3
80	2.0	20000.0	.323	3447.2	.275	2174.1	.187	1215.7
160	2.0	20000.0	.250	2565.5	.188	1733.3	.119	976.0
5	4.0	20000.0	11.383	7210.0	11.383	7210.0	1.345	2384.1
10	4.0	20000.0	2.124	10787.4	1.853	6984.0	1.278	1923.5
20	4.0	20000.0	2.509	4316.0	1.946	2762.6	1.065	1366.4
40	4.0	20000.0	1.356	5245.4	1.105	2717.0	.708	803.0
80	4.0	20000.0	.898	2715.7	.750	1513.0	.538	494.1
160	4.0	20000.0	.556	1517.9	.390	888.6	.231	460.7
5	8.0	20000.0	*8.075	3630.2	*8.075	3630.2	4.817	1357.9
10	8.0	20000.0	6.466	3679.6	5.133	2740.0	2.575	841.6
20	8.0	20000.0	3.893	2893.1	3.471	1759.6	1.978	624.2
40	8.0	20000.0	2.139	1166.2	1.812	764.9	1.378	338.6
80	8.0	20000.0	1.845	1013.2	1.364	602.3	.973	368.7
160	8.0	20000.0	1.186	580.0	1.006	298.8	.682	144.8

STANDARD ERROR

******MEDIAN RANKS******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	40000.0	.680	5409179.8	.680	5409179.8	.229	52061.7
10	.5	40000.0	.430	1319197.6	.352	424512.8	.140	36064.2
20	.5	40000.0	.346	478201.3	.268	130973.1	.087	24065.1
40	.5	40000.0	.181	469644.3	.157	97085.7	.092	11177.1
80	.5	40000.0	.100	29732.7	.075	17214.4	.047	10053.3
160	.5	40000.0	.067	34843.0	.052	18762.3	.039	7646.1
5	1.0	40000.0	3.544	116388.8	3.544	116388.8	.417	19065.9
10	1.0	40000.0	.772	65283.3	.600	29591.9	.285	8948.9
20	1.0	40000.0	.472	55380.8	.358	19400.4	.185	9565.2
40	1.0	40000.0	.294	103037.4	.245	31355.0	.176	8818.1
80	1.0	40000.0	.176	14627.2	.146	6613.2	.112	3880.2
160	1.0	40000.0	.153	11464.8	.125	6546.4	.076	3088.7
5	2.0	40000.0	12.038	28133.4	12.038	28133.4	1.591	9880.9
10	2.0	40000.0	1.320	21814.6	1.245	15121.0	.713	6815.2
20	2.0	40000.0	1.279	16104.0	.665	9642.7	.313	4564.9
40	2.0	40000.0	.665	7139.4	.620	4645.4	.317	2417.0
80	2.0	40000.0	.404	9906.3	.352	4795.1	.252	2533.4
160	2.0	40000.0	.343	5128.3	.292	3254.2	.190	1629.3
5	4.0	40000.0	9.517	41023.0	9.517	41023.0	2.203	4497.6
10	4.0	40000.0	1.774	16396.6	1.539	11535.6	1.043	3908.5
20	4.0	40000.0	1.641	7861.1	1.334	4376.2	.831	1516.2
40	4.0	40000.0	1.534	4214.5	1.366	2758.9	.686	1210.6
80	4.0	40000.0	.788	3794.4	.645	2418.4	.417	1125.1
160	4.0	40000.0	.523	3333.1	.413	1865.0	.276	733.3
5	8.0	40000.0	23.824	4199.7	23.824	4199.7	5.203	2008.4
10	8.0	40000.0	8.477	4951.3	5.506	3639.9	2.053	1692.4
20	8.0	40000.0	3.544	3389.1	2.470	2457.0	1.367	1259.5
40	8.0	40000.0	1.663	2001.8	1.594	1436.1	1.255	932.8
80	8.0	40000.0	1.215	1561.2	.918	1054.5	.637	570.8
160	8.0	40000.0	1.198	1135.0	.892	667.0	.598	321.6

STANDARD ERROR

******MEDIAN RANKS******

****VARIABLES****			PERCENTAGE SUSPENSION					
SAMPLE			60%		40%		0%	
SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	80000.0	7.390	29910863.2	7.390	29910863.2	.234	82799.2
10	.5	80000.0	.209	37858312.6	.178	4364996.5	.124	48814.0
20	.5	80000.0	.203	1003922.5	.160	273959.9	.103	61998.9
40	.5	80000.0	.142	109270.5	.120	53452.4	.069	26439.0
80	.5	80000.0	.107	94227.6	.091	50023.6	.057	24312.4
160	.5	80000.0	.059	37390.5	.045	22030.7	.031	12274.4
5	1.0	80000.0	.866	195172.1	.868	195172.1	.879	35504.5
10	1.0	80000.0	.568	512043.2	.566	226288.4	.309	28124.0
20	1.0	80000.0	.342	108812.4	.258	58619.0	.209	24216.7
40	1.0	80000.0	.529	94582.6	.369	40115.2	.179	16185.8
80	1.0	80000.0	.249	42919.1	.194	23175.6	.118	13167.3
160	1.0	80000.0	.202	58800.3	.162	23877.5	.106	5830.1
5	2.0	80000.0	28.943	161076.5	28.943	161076.5	.760	17846.5
10	2.0	80000.0	1.276	56147.0	.972	35542.1	.499	13678.6
20	2.0	80000.0	.349	31398.3	.617	16375.5	.417	79[illegible]0.4
40	2.0	80000.0	.700	30452.0	.592	16432.2	.357	6491.8
80	2.0	80000.0	.510	23486.1	.417	12563.6	.246	4725.6
160	2.0	80000.0	.388	15841.2	.331	8815.5	.203	2423.8
5	4.0	80000.0	49.804	40040.2	49.804	40040.2	2.214	8941.4
10	4.0	80000.0	2.202	36140.8	1.990	23760.5	1.350	6635.4
20	4.0	80000.0	1.436	11325.8	1.063	7574.4	.598	3411.7
40	4.0	80000.0	1.062	14607.6	.852	7936.8	.653	3093.0
80	4.0	80000.0	.841	7495.4	.700	4723.8	.455	2696.8
160	4.0	80000.0	.626	4288.8	.511	2557.0	.323	1708.7
5	8.0	80000.0	31.222	13820.2	31.222	13820.2	3.588	6532.2
10	8.0	80000.0	6.865	8436.7	6.618	5591.1	2.639	3022.7
20	8.0	80000.0	3.147	9277.5	2.240	5683.8	1.350	2068.2
40	8.0	80000.0	2.102	3953.7	1.818	2761.8	1.165	1518.3
80	8.0	80000.0	1.229	3727.6	1.051	2410.6	.724	1226.8
160	8.0	80000.0	1.454	2883.4	1.142	1326.8	.732	706.7

STANDARD ERROR

******MEDIAN RANKS******

****VARIABLES****			PERCENTAGE SUSPENSION					
			60%		40%		0%	
SAMPLE SIZE	BETA	THETA	BETA	THETA	BETA	THETA	BETA	THETA
5	.5	160000.0	3.311	*1248581.1	3.311	*1248581.1	.182	198376.8
10	.5	160000.0	.476	2227641.6	.312	861395.6	.199	91535.8
20	.5	160000.0	.241	4906581.5	.181	862046.8	.143	106242.0
40	.5	160000.0	.192	1213659.2	.149	333768.1	.090	75348.1
80	.5	160000.0	.132	532660.6	.084	135995.0	.046	42007.3
160	.5	160000.0	.055	69208.6	.052	39886.7	.040	27296.7
5	1.0	160000.0	9.489	310432.0	9.489	310432.0	.698	79766.8
10	1.0	160000.0	2.415	157754.9	1.341	98974.0	.366	50595.9
20	1.0	160000.0	.597	361878.0	.349	131644.4	.212	51286.0
40	1.0	160000.0	.264	95373.6	.182	46779.3	.109	17825.1
80	1.0	160000.0	.225	72302.2	.145	41192.2	.108	22017.9
160	1.0	160000.0	.126	48161.1	.095	25363.5	.059	13031.1
5	2.0	160000.0	68.292	60956.4	68.292	60956.4	.699	35750.6
10	2.0	160000.0	1.464	95124.7	1.315	59622.5	.555	23418.2
20	2.0	160000.0	1.211	84338.1	1.063	44873.2	.490	23704.2
40	2.0	160000.0	.611	53747.4	.529	32593.1	.340	11187.2
80	2.0	160000.0	.371	28052.7	.304	16858.7	.199	7759.7
160	2.0	160000.0	.330	27346.7	.268	15539.4	.172	6360.1
5	4.0	160000.0	28.801	38207.7	28.801	38207.7	2.217	21546.1
10	4.0	160000.0	2.528	34894.1	2.414	28262.5	.755	11456.0
20	4.0	160000.0	2.658	25835.9	1.933	17444.1	.978	9693.2
40	4.0	160000.0	1.045	24249.5	.848	14363.1	.591	7742.2
80	4.0	160000.0	.941	16293.1	.666	7912.7	.342	4500.6
160	4.0	160000.0	.521	10078.9	.351	6025.2	.251	3225.0
5	8.0	160000.0	56.559	23109.8	56.589	23109.8	6.021	7973.0
10	8.0	160000.0	9.407	19404.3	3.983	14882.3	2.439	5037.5
20	8.0	160000.0	3.832	18580.4	3.292	11496.2	2.123	4320.3
40	8.0	160000.0	2.677	10456.0	2.437	5987.8	1.626	3096.3
80	8.0	160000.0	1.725	8367.5	1.379	5343.2	.793	2848.6
160	8.0	160000.0	1.316	6064.5	1.101	3655.2	.752	1765.9

APPENDIX IV

RESULTS OF SIMULATION STUDY (PLOTTED)

COMPARISON OF STANDARD ERRORS
EXPERIENCED IN CALCULATING $\hat{\beta}$

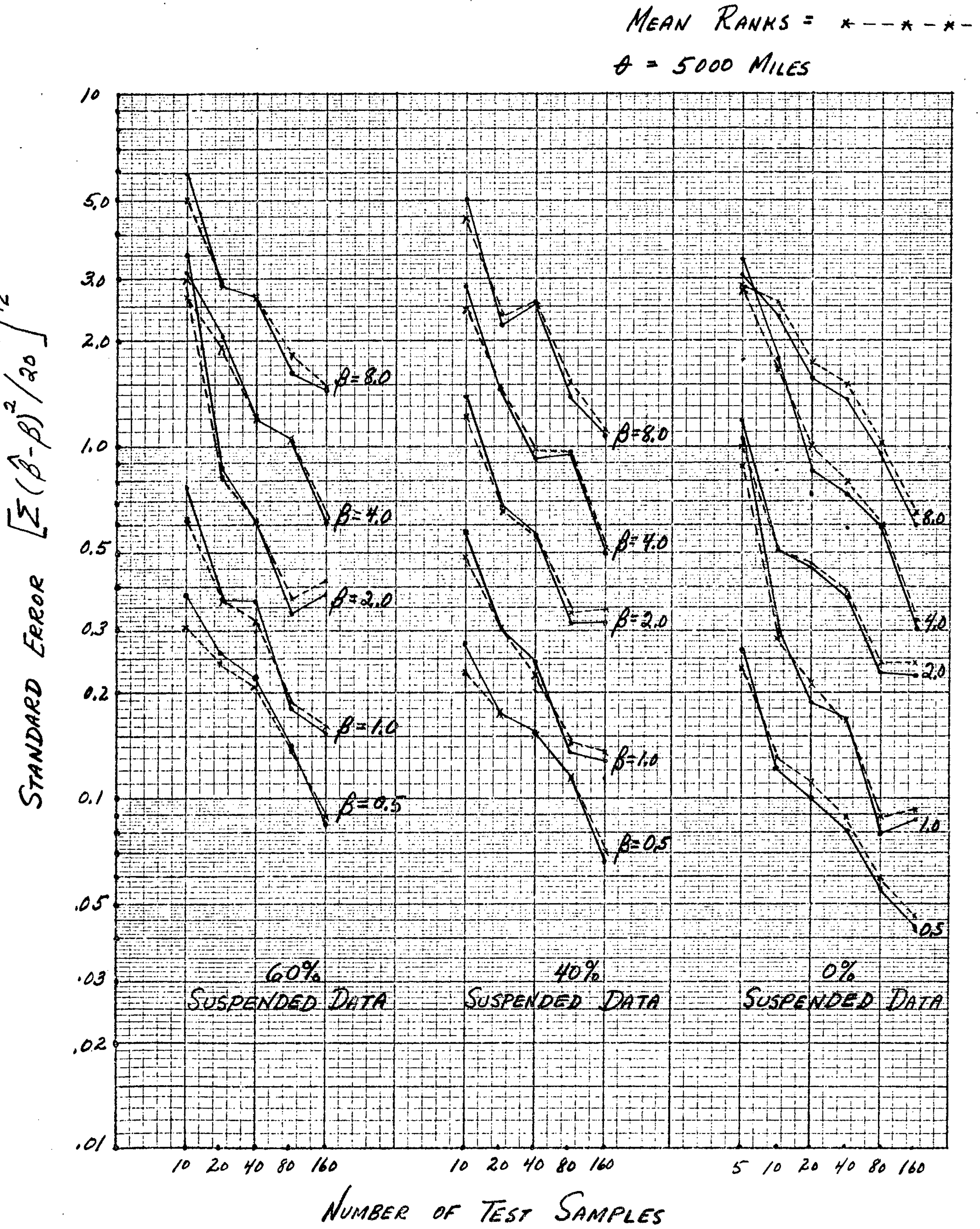
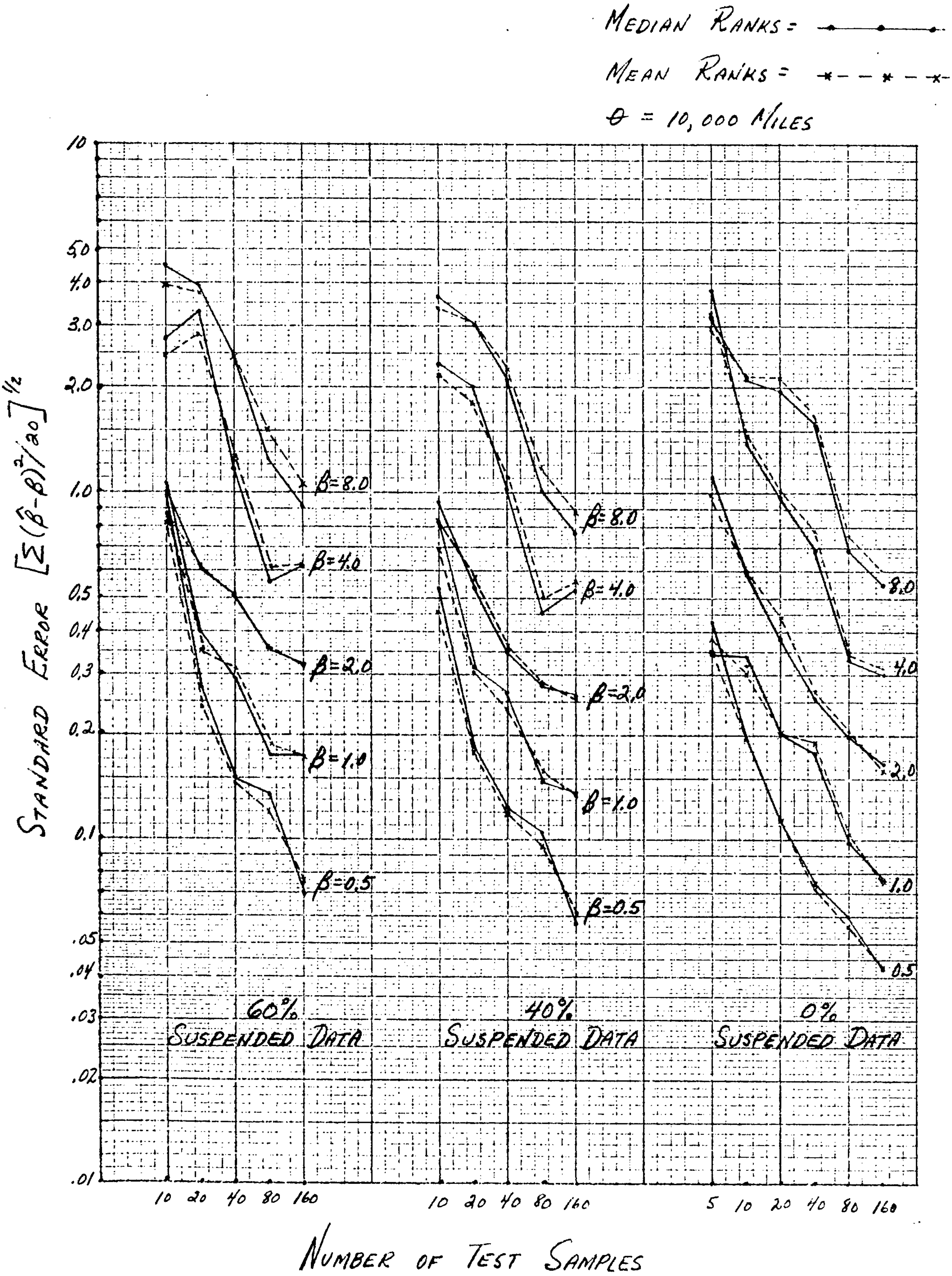
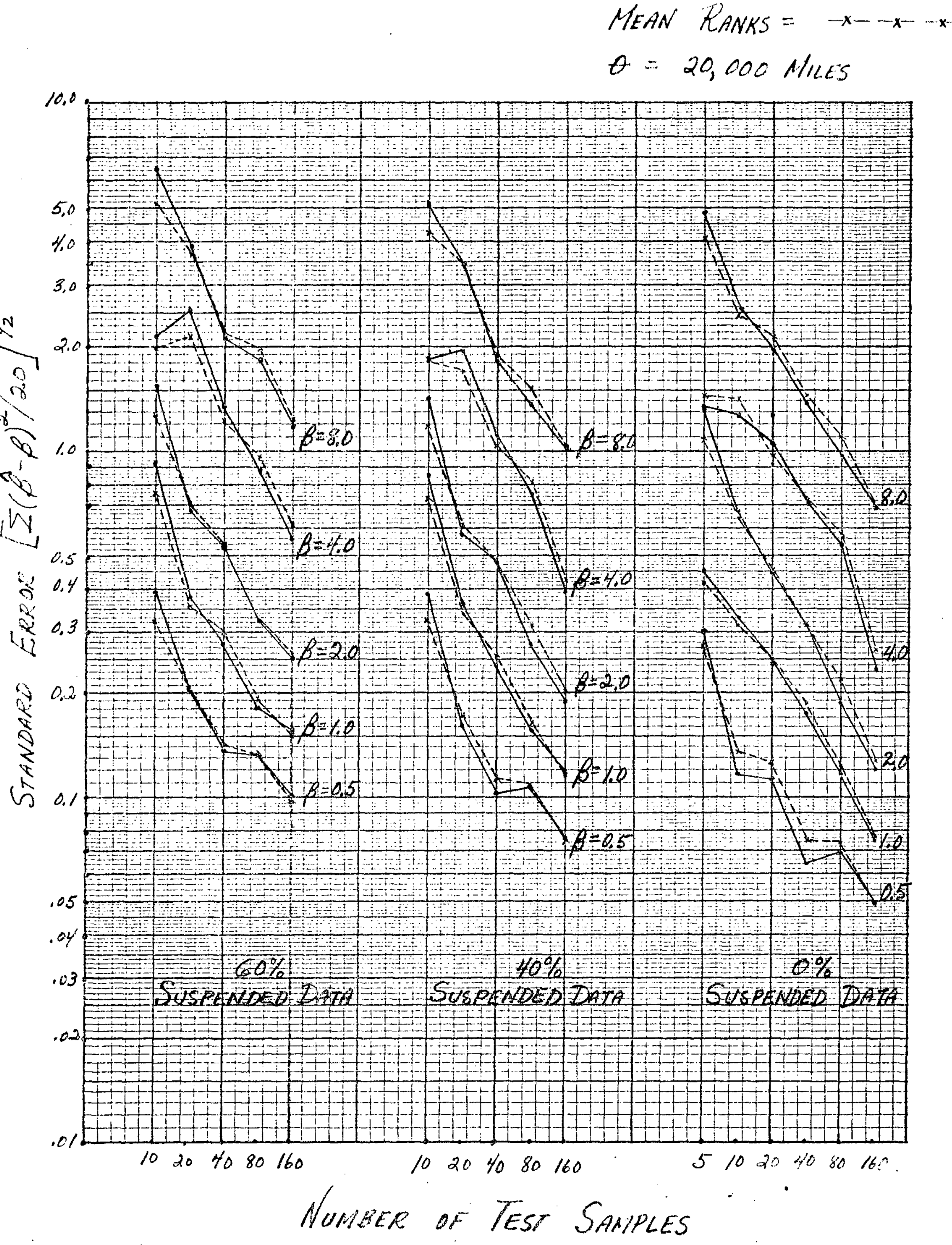

MEAN RANKS = *--*-*-
θ = 5000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\beta}-\beta)^2/20\right]^{1/2}$
10
5.0
3.0
2.0
1.0
0.5
0.3
0.2
0.1
.05
.03
.02
.01
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
8.0
4.0
2.0
1.0
0.5
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 14



MEDIAN RANKS =
MEAN RANKS =
θ = 10,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\beta}-\beta)^2/20\right]^{1/2}$
10
5.0
4.0
3.0
2.0
1.0
0.5
0.4
0.3
0.2
0.1
.05
.04
.03
.02
.01
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
8.0
4.0
2.0
1.0
0.5
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 15



MEAN RANKS = —x— —x— —x-
θ = 20,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\beta}-\beta)^2/20\right]^{1/2}$
10.0
5.0
4.0
3.0
2.0
1.0
0.5
0.4
0.3
0.2
0.1
.05
.04
.03
.02
.01
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
8.0
4.0
2.0
1.0
0.5
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 16

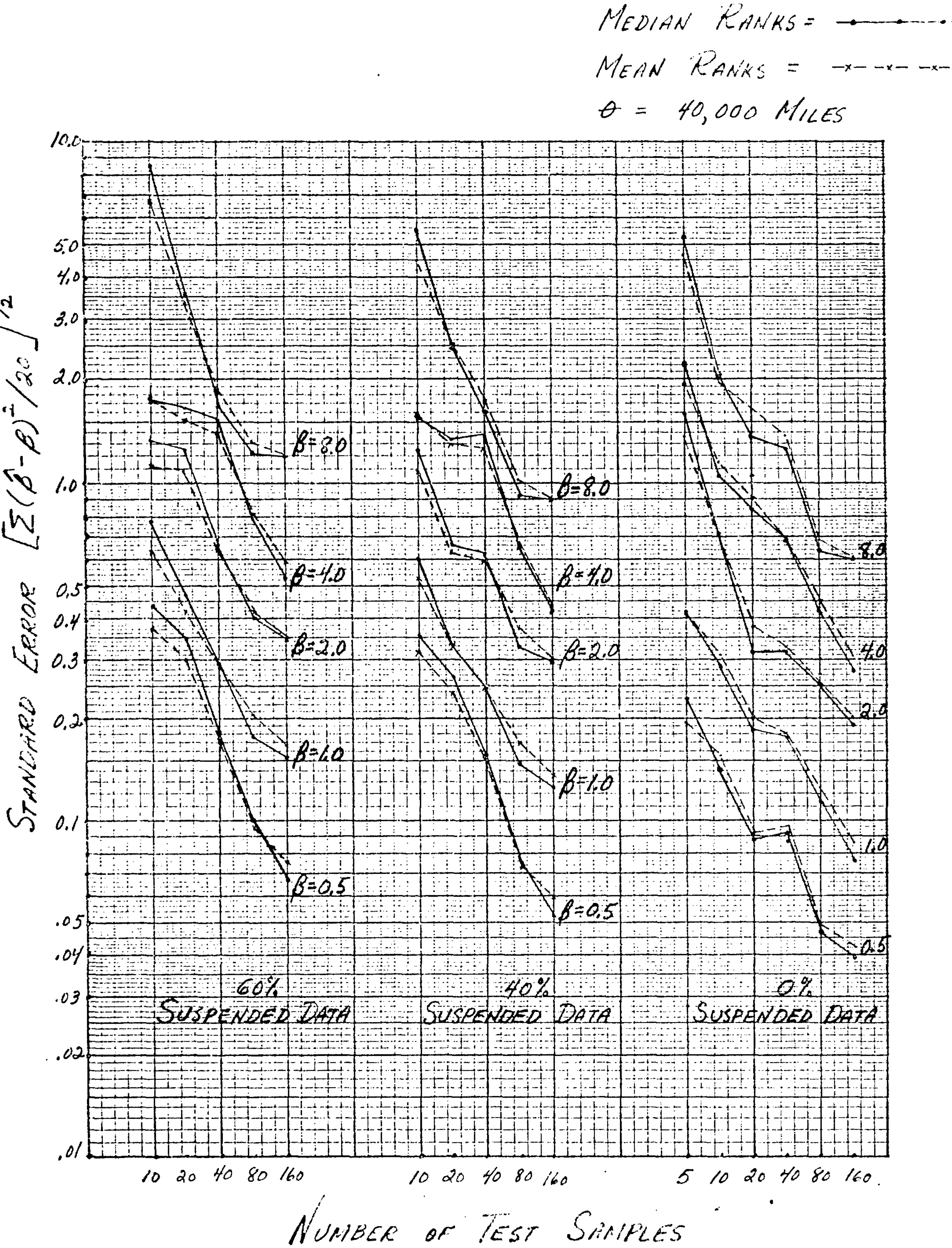
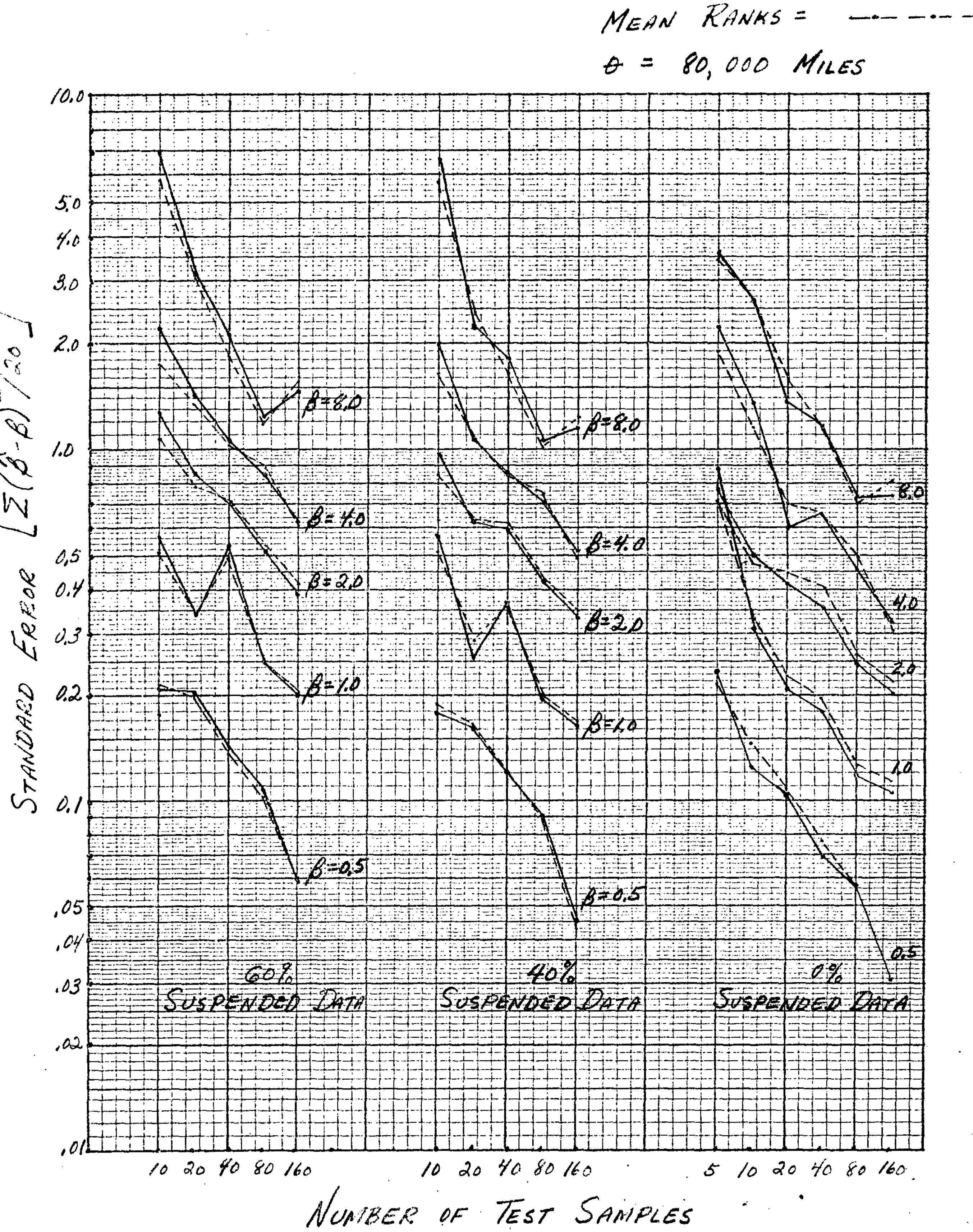

MEDIAN RANKS =
MEAN RANKS =
θ = 40,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\beta}-\beta)^2/20\right]^{1/2}$
10.0
5.0
4.0
3.0
2.0
1.0
0.5
0.4
0.3
0.2
0.1
.05
.04
.03
.02
.01
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
8.0
4.0
2.0
1.0
0.5
60%
SUSPENDED DATA
40%
SUSPENDED DATA
0%
SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 17



MEAN RANKS = —·—·—
θ = 80,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\beta}-\beta)^2/20\right]^{1/2}$
10.0
5.0
4.0
3.0
2.0
1.0
0.5
0.4
0.3
0.2
0.1
.05
.04
.03
.02
.01
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
60% SUSPENDED DATA
β=8.0
β=4.0
β=2.0
β=1.0
β=0.5
40% SUSPENDED DATA
8.0
4.0
2.0
1.0
0.5
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 18

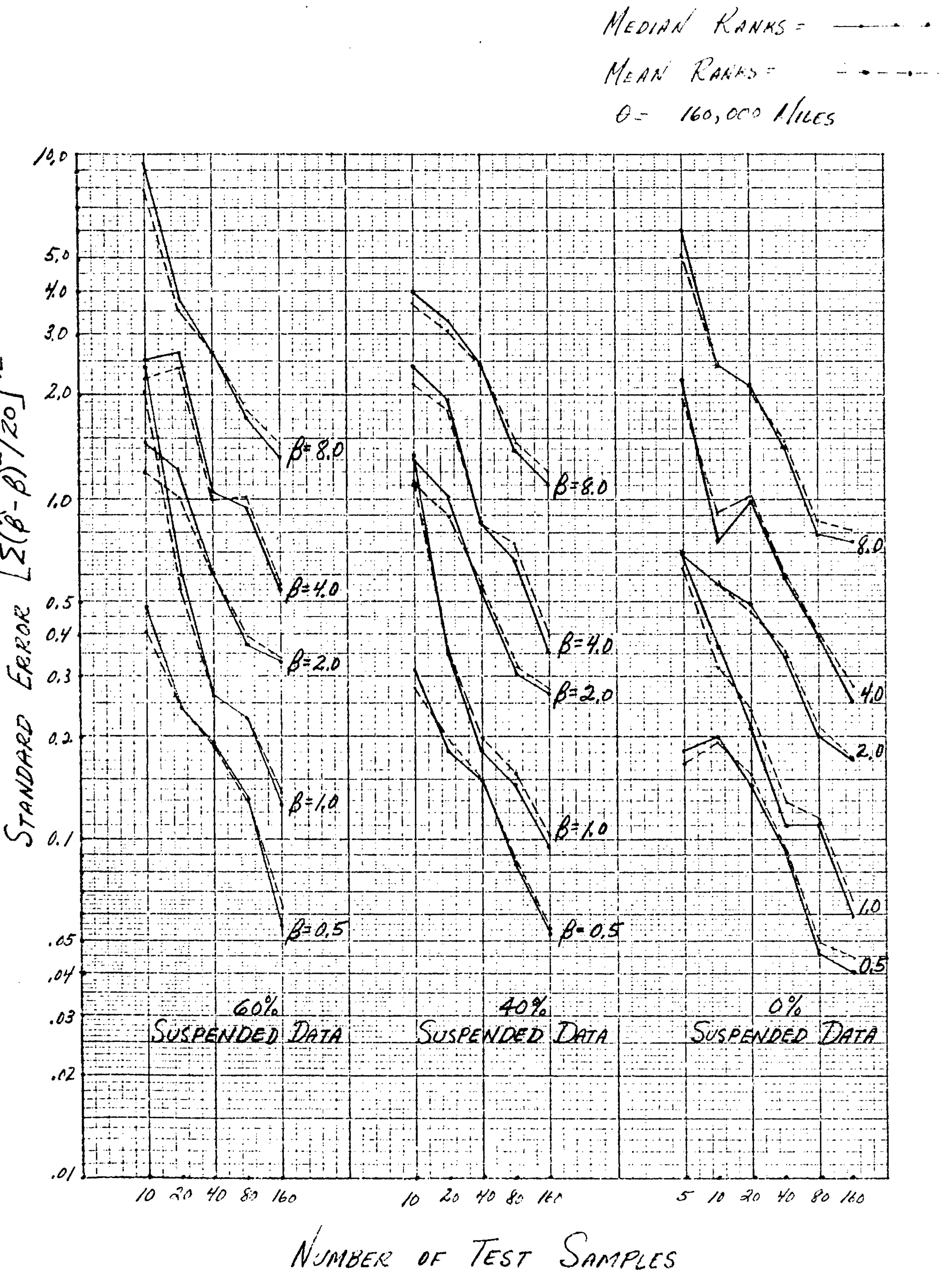

MEDIAN RANKS =
MEAN RANKS =
θ = 160,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\beta}-\beta)^2/20\right]^{1/2}$
10.0
5.0
4.0
3.0
2.0
1.0
0.5
0.4
0.3
0.2
0.1
.05
.04
.03
.02
.01
β = 8.0
β = 4.0
β = 2.0
β = 1.0
β = 0.5
β = 8.0
β = 4.0
β = 2.0
β = 1.0
β = 0.5
8.0
4.0
2.0
1.0
0.5
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 19

COMPARISON OF STANDARD ERRORS

EXPERIENCED IN CALCULATING $\hat{\theta}$

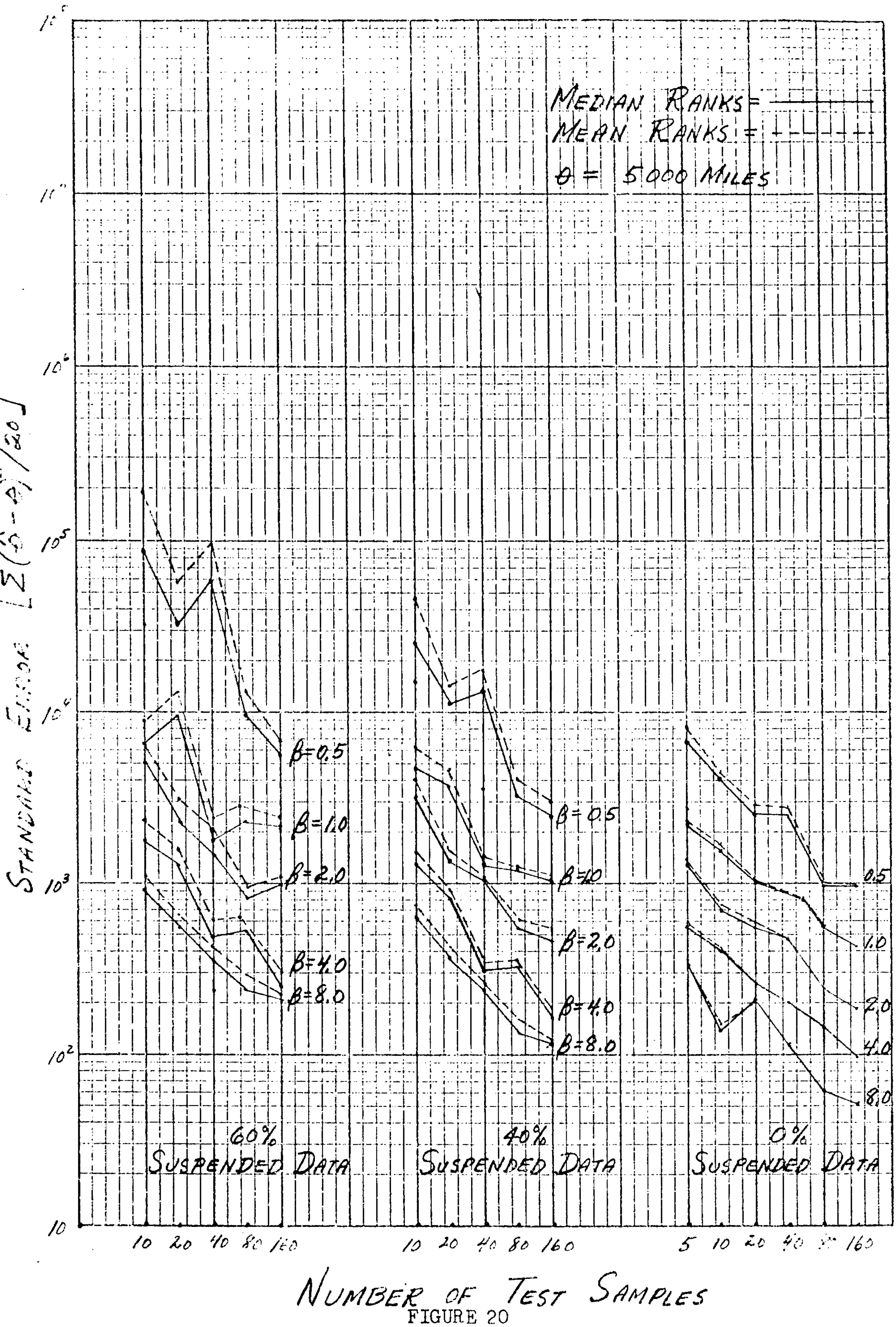
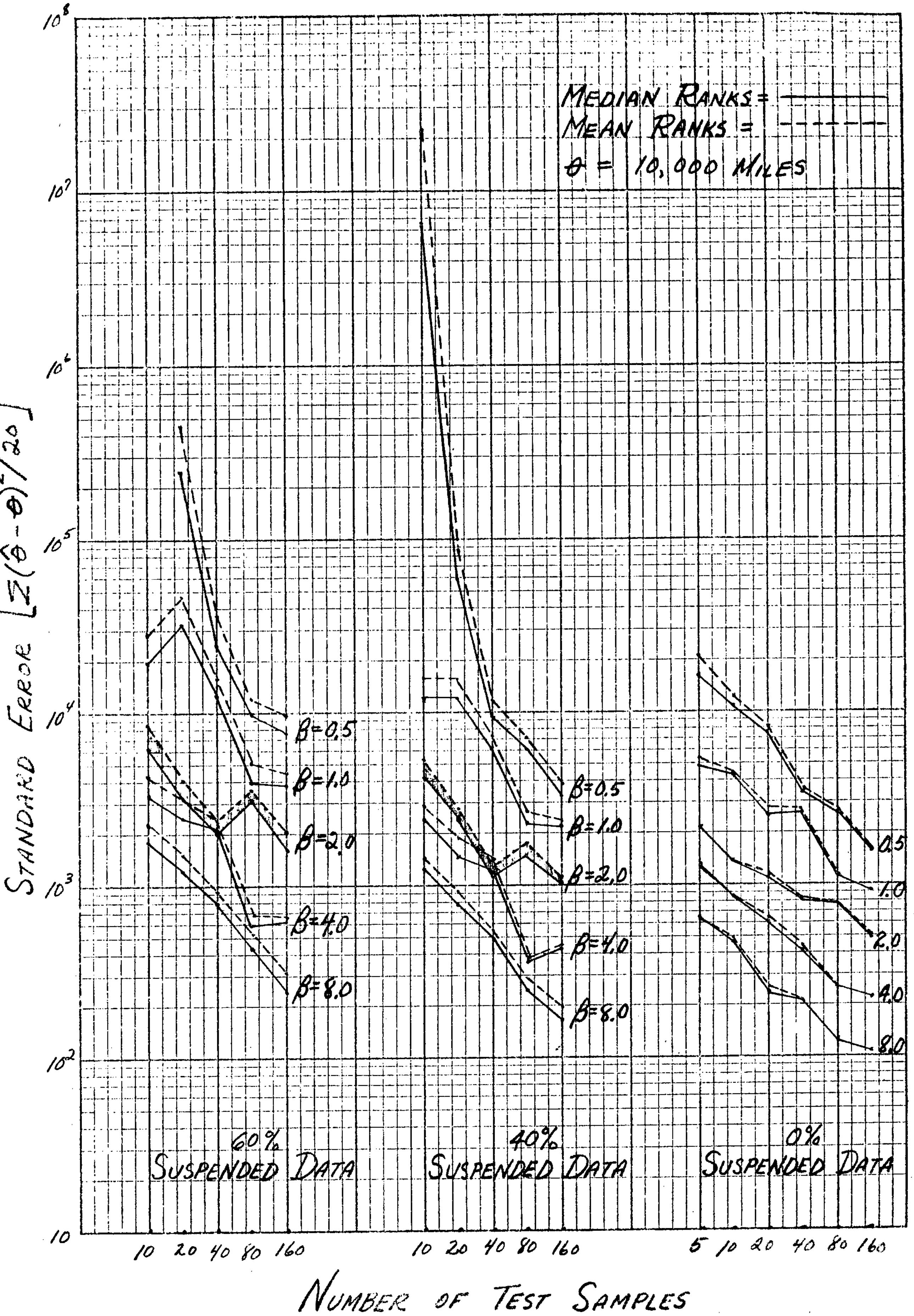
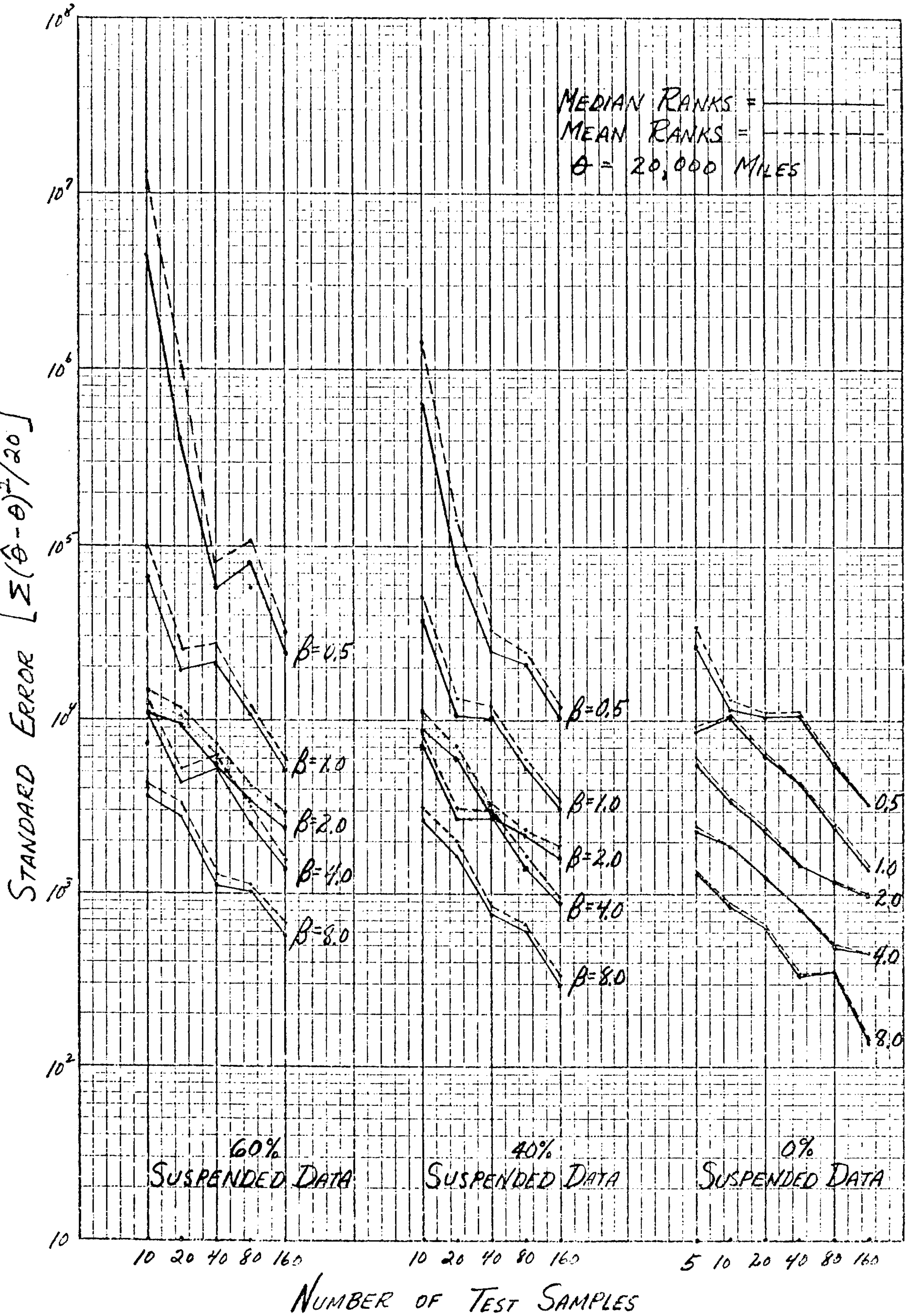

MEDIAN RANKS =
MEAN RANKS =
θ = 5000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\theta}-\theta)^2/20\right]^{1/2}$
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
β=0.5
β=1.0
β=2.0
β=4.0
β=8.0
β=0.5
β=1.0
β=2.0
β=4.0
β=8.0
0.5
1.0
2.0
4.0
8.0
60%
SUSPENDED DATA
40%
SUSPENDED DATA
0%
SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 20



MEDIAN RANKS =
MEAN RANKS =
θ = 10,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\theta}-\theta)^2/20\right]^{1/2}$
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
β=0.5
β=1.0
β=2.0
β=4.0
β=8.0
0.5
1.0
2.0
4.0
8.0
60%
SUSPENDED DATA
40%
SUSPENDED DATA
0%
SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 21



MEDIAN RANKS =
MEAN RANKS =
θ = 20,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\theta}-\theta)^2/20\right]^{1/2}$
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
β=0.5
β=1.0
β=2.0
β=4.0
β=8.0
0.5
1.0
2.0
4.0
8.0
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 22

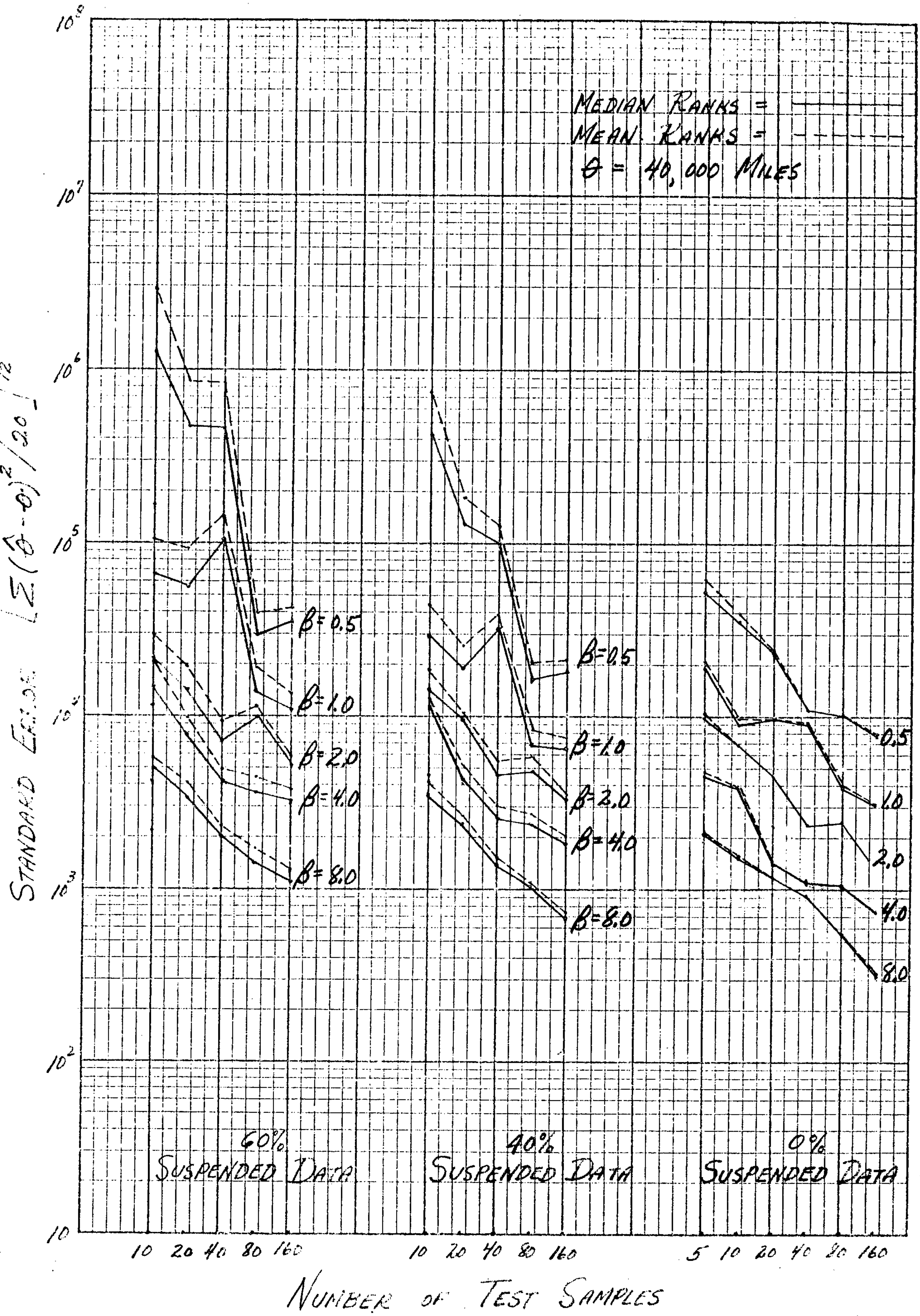
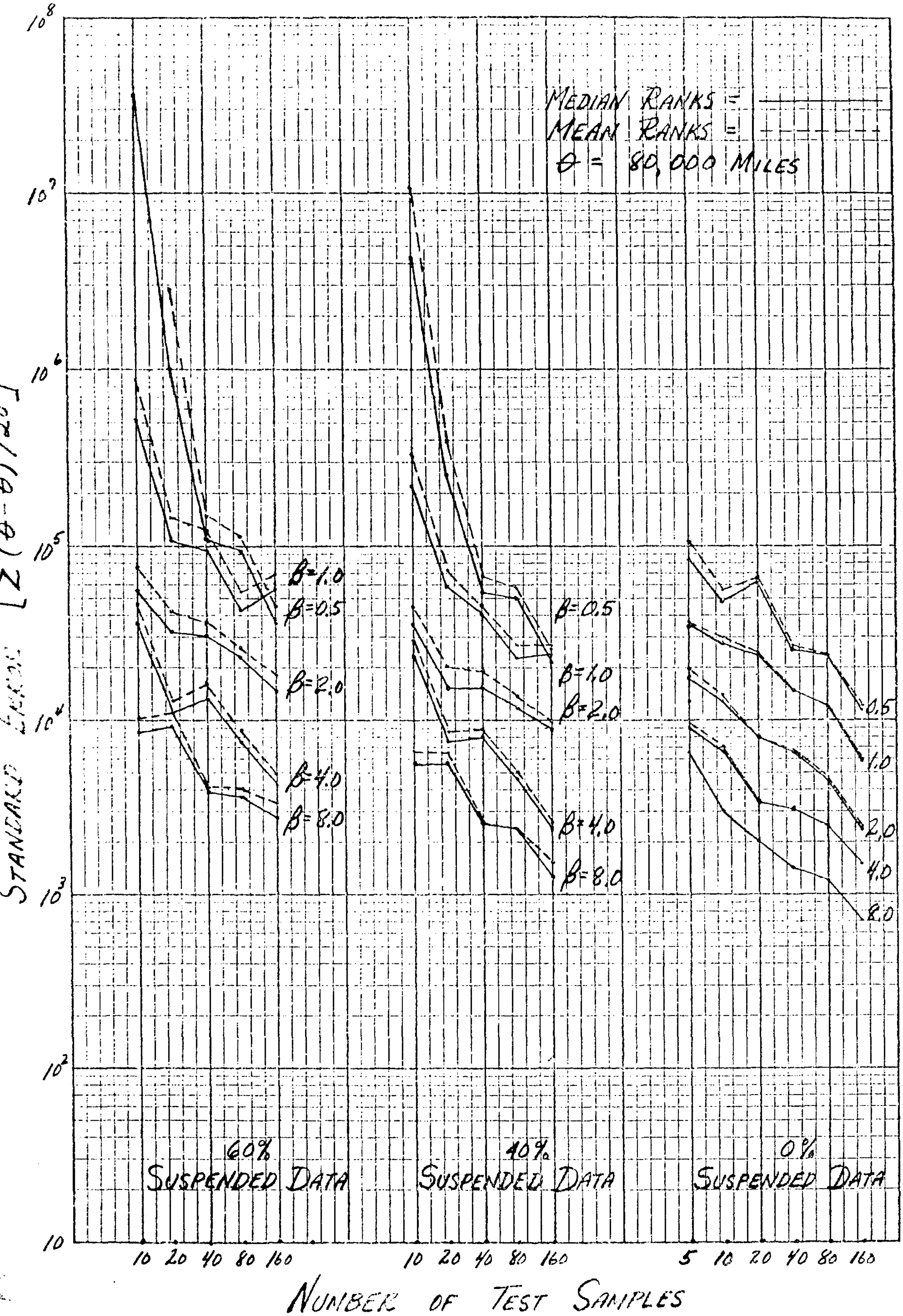

MEDIAN RANKS =
MEAN RANKS =
θ = 40,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\theta}-\theta)^2/20\right]^{1/2}$
β = 0.5
β = 1.0
β = 2.0
β = 4.0
β = 8.0
0.5
1.0
2.0
4.0
8.0
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 23



MEDIAN RANKS =
MEAN RANKS =
θ = 80,000 MILES
STANDARD ERROR $\left[\Sigma(\hat{\theta}-\theta)^2/20\right]^{1/2}$
NUMBER OF TEST SAMPLES
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA

FIGURE 24

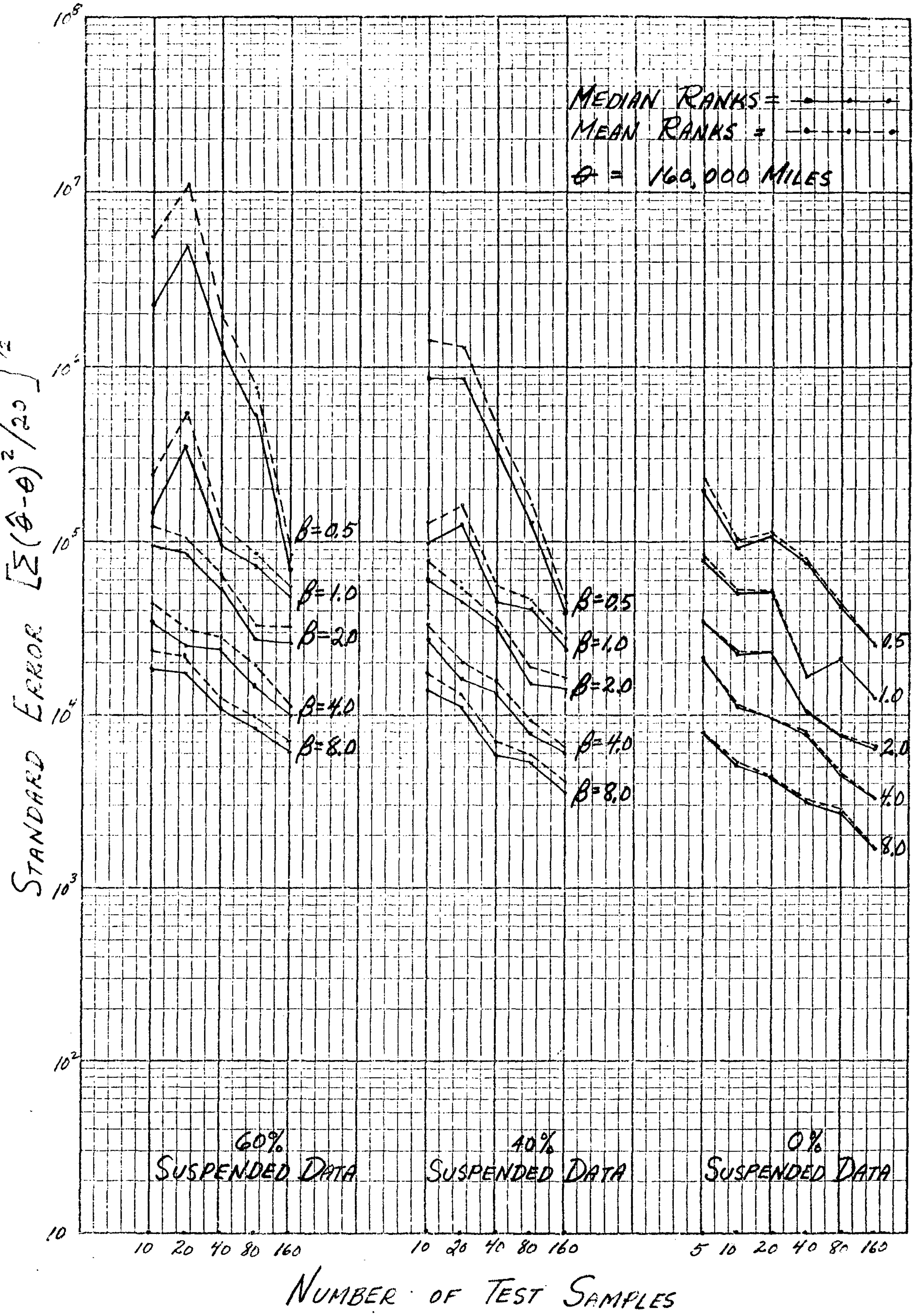

MEDIAN RANKS =
MEAN RANKS =
θ = 160,000 MILES
STANDARD ERROR $[\Sigma(\hat{\theta}-\theta)^2/20]^{1/2}$
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
β=0.5
β=1.0
β=2.0
β=4.0
β=8.0
0.5
1.0
2.0
4.0
8.0
60% SUSPENDED DATA
40% SUSPENDED DATA
0% SUSPENDED DATA
10 20 40 80 160
10 20 40 80 160
5 10 20 40 80 160
NUMBER OF TEST SAMPLES

FIGURE 25

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Chief Scientist, AMSTA-CL	1
Systems Development Division, AMSTA-RE	1
Transport Vehicle Branch, AMSTA-RFB	1
Foreign Intelligence Office, AMSTA-RI	1
Technical Data Division, AMSTA-RS	1
Surface Mobility Division, AMSTA-RU	1
Frame, Suspension & Track Functions, AMSTA-RUT	1
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M60 Tank, AMCPM-M60	1
Mechanized Infantry Combat Vehicle, AMCPM-MCV	2
Armored Reconnaissance Scout Vehicle, AMCPM-RSV	1
M561/XM705 Trucks, AMCPM-GG	1
XM-1 Tank System, AMCPM-GCM	1
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13. ABSTRACT

Estimates of the Weibull distribution parameters were made employing the mean ranks estimator; the estimates were repeated using the median ranks estimator. These estimates were compared to known values of the Weibull distribution parameters. This made it possible to compare the results obtained using either estimator (mean ranks or median ranks) and to determine the relative merits of using either estimator. This study made use of a digital computer and employed Monte-Carlo techniques to simulate Weibull distributed failure times. These failure times may represent tank-automotive component failures.

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